EXAM P QUESTIONS OF THE WEEK

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Week of July 3/06

A loaded six-sided die has the following probability function:
$P(X=1)=P(X=3)=P(X=5)=\frac{1}{9}$,
$P(X=2)=P(X=4)=P(X=6)=\frac{2}{9}$.

The die is tossed repeatedly until the outcome is 1, 2 or 3.
The first 1, 2 or 3 is the random variable Y. Find the variance of Y.

The solution can be found below.

Week of July 3/06 - Solution

Y can be thought of as the conditional distribution of X given that X is not 4, 5 or 6.

The probability function of Y is

$$P(Y=1) = P(X=1|X \neq 4,5,6) = \frac{P(X=1)}{P(X \neq 4,5,6)} = \frac{1/9}{4/9} = \frac{1}{4}\,,$$
$$P(Y=2) = P(X=2|X \neq 4,5,6) = \frac{P(X=2)}{P(X \neq 4,5,6)} = \frac{2/9}{4/9} = \frac{1}{2}\,,$$
$$P(Y=3) = P(X=3|X \neq 4,5,6) = \frac{P(X=3)}{P(X \neq 4,5,6)} = \frac{1/9}{4/9} = \frac{1}{4}\ .$$

$$E[Y] = (1)(\tfrac{1}{4}) + (2)(\tfrac{1}{2}) + (3)(\tfrac{1}{4}) = 2\ ,$$
$$E[Y^2] = (1^2)(\tfrac{1}{4}) + (2^2)(\tfrac{1}{2}) + (3^2)(\tfrac{1}{4}) = \frac{9}{2}\ .$$

$$Var[Y] = E[Y^2] - (E[Y])^2 = \frac{9}{2} - 2^2 = \frac{1}{2}\ .$$