EXAM C QUESTIONS OF THE WEEK

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Week of June 12/06

You are given the following random sample of 15 times of failure for a mechanical device:

$$5\,,\ 6\,,\ 6\,,\ 8\,,\ 9\,,\ 11\,,\ 13\,,\ 14\,,\ 18\,,\ 18\,,\ 18\,,\ 20\,,\ 21\,,\ 24\,,\ 28$$

$A =$ smoothed empirical estimate of the 80-th percentile of the time until failure of the device, and

$B =$ smoothed empirical estimate of the 80-th percentile of the time until failure of the device given that the device survives to at least time 12.

Find $A - B$.

Solution can be found below.

Week of June 12/06 - Solution

There are 15 data points, so the smoothed empirical percentiles assigned to the data points are
$\frac{1}{n+1}$, $\frac{2}{n+1}$, . . . :

$5 \quad , \quad 6 \quad , \quad 6 , \quad 8 , 9 , 11 , 13 , 14 , 18 , 18 , 18 , 20 , 21 , 24 , 28$
$\frac{1}{16}$, $\frac{2}{16}$, $\frac{3}{16}$, . . . ,

We see that 12-th data point $x = 20$ is the smoothed empirical percentile $\frac{12}{16} = .75$ (75-th percentile), and the 13-th data point $x = 21$ is the smoothed empirical percentile $\frac{13}{16} = .8125$ (81.25-th percentile). Since .8 is .8 of the way from $.75$ to $.8125$, the smoothed empirical estimate of the 80-th percentile is .8 of the way from 20 to 21, which is $A = 20.8$.

Given that the device survives to at least time 12, the sample of failure times is
$13 , 14 , 18 , 18 , 18 , 20 , 21 , 24 , 28$ (9 data points).
The smoothed empirical percentiles assigned to these data points are
$13 , 14 , 18 , 18 , 18 , 20 , 21 , 24 , 28$
$\frac{1}{10}$, $\frac{2}{10}$, . . .
The smoothed empirical estimate of the 80th percentile is the 8-th data point, which is $B = 24$.

$A - B = 20.8 - 24 = -3.2$.