EXAM M QUESTIONS OF THE WEEK

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Week of June 19/06

The force of mortality is given to be $\mu(y) = \frac{3}{100+y}$ for $y > 0$.

$A =$ median future lifetime for someone at age $y = 50$, and
$B =$ mean future lifetime for someone at age $y = 50$.

Find $A - B$.

The solution can be found below.

Week of June 19/06 - Solution

If $y = 50$, then ${}_tp_{50} = e^{-\int_0^t \mu(50+s)\,ds} = e^{-\int_0^t (\frac{3}{100+50+s})\,ds} = e^{-3 \cdot ln(\frac{150+t}{150})} = (\frac{150}{150+t})^3$.

The median future lifetime A is the 50-th percentile of the survival time random variable, so that ${}_Aq_{50} = .5$. Therefore, $(\frac{150}{150+A})^3 = .5$, from which we get $A = 39$.

The mean future lifetime is $B = \mathring{e}_{50} = \int_0^\infty {}_tp_{50}\,dt = \int_0^\infty (\frac{150}{150+t})^3\,dt = \frac{150}{2} = 75$.

$A - B = -36$.