EXAM M QUESTIONS OF THE WEEK

S. Broverman, 2006

Week of June 26/06

A select and ultimate life table with a one year select period satisfies UDD, with $q_{[x]} = .1$ and $A_{x+1} = .4$. Suppose that $q_{[x]}$ is changed to .05, but all other mortality probabilities are unchanged. Find the change in $\overline{A}_{[x]}$ if $i = .08$.

The solution can be found below.

Week of June 26/06 - Solution

Using the original value of $q_{[x]} = .1$, we have $\overline{A}_{[x]} = \overline{A}_{[x]:\overline{1|}}^{1} + vp_{[x]} \cdot \overline{A}_{x+1}$.

Under UDD, we have $\overline{A}_{[x]:\overline{1|}}^{1} = \frac{i}{\delta} \cdot A_{[x]:\overline{1|}}^{1} = \frac{.08}{ln\,1.08} \cdot vq_{[x]} = .096249$,

and $\overline{A}_{x+1} = \frac{i}{\delta} \cdot A_{x+1} = \frac{.08}{ln\,1.08} \cdot (.4) = .415795$.

Using the original value of $q_{[x]} = .1$, we have

$\overline{A}_{[x]} = .096249 + \frac{.9}{1.08} \cdot (.415796) = .4427$.

Using the new value of $q_{[x]} = .05$, we have $\overline{A}_{[x]:\overline{1|}}^{1} = \frac{i}{\delta} \cdot A_{[x]:\overline{1|}}^{1} = \frac{.08}{ln\,1.08} \cdot vq_{[x]} = .048124$,

and $\overline{A}_{x+1} = .415795$ as before.

Using the new value of $q_{[x]} = .05$, we have

$\overline{A}_{[x]} = .048124 + \frac{.95}{1.08} \cdot (.415796) = .4139$.

The change in $\overline{A}_{[x]}$ is a decrease of .0288.

Alternatively,

$\overline{A}_{[x]}^{New} - \overline{A}_{[x]}^{Old} = \overline{A}_{[x]:\overline{1|}}^{1\,New} + vp_{[x]}^{New} \cdot \overline{A}_{x+1}^{New} - [\overline{A}_{[x]:\overline{1|}}^{1\,Old} + vp_{[x]}^{Old} \cdot \overline{A}_{x+1}^{Old}]$

$= \frac{i}{\delta} \cdot v(q_{[x]}^{New} - q_{[x]}^{Old}) + v(p_{[x]}^{New} - p_{[x]}^{Old}) \cdot \overline{A}_{x+1}$ (since $\overline{A}_{x+1}^{New} = \overline{A}_{x+1}^{Old}$).

$= \frac{.08}{ln\,1.08} \cdot \frac{1}{1.08} \cdot (.05 - .1) + \frac{1}{1.08} \cdot (.95 - .9)(.415795) = -.0289$.