EXAM M QUESTIONS OF THE WEEK

S. Broverman, 2006

Week of March 27/06

The ground up loss random variable X for a health insurance policy in 2006 is modeled with an exponential distribution with mean 1000. An insurance policy pays the loss above an ordinary deductible of 100, with a maximum annual payment of 500. The ground up loss random variable is expected to be 5% larger in 2007, but the insurance in 2007 has the same deductible and maximum payment as in 2006. Find the percentage increase in the expected cost per payment from 2006 to 2007.

The solution can be found below.

Week of March 27/06 - Solution

In 2006 the deductible is $d = 100$ and the maximum covered loss is $u = 600$ (policy limit of $u - d = 500$). The expected cost per loss in 2006 is $E[X \wedge u] - E[X \wedge d]$,
and the expected cost per payment is $\frac{E[X\wedge u]-E[X\wedge d]}{1-F_X(d)}$.
For the exponential distribution with mean θ, we have
$F_X(x) = 1 - e^{-x/\theta}$ and $E[X \wedge x] = \theta(1 - e^{-x/\theta})$.
The expected cost per payment in 2006 is
$\frac{E[X\wedge 600]-E[X\wedge 100]}{1-F_X(100)} = \frac{1000(1-e^{-600/1000})-1000(1-e^{-100/1000})}{1-(1-e^{-100/1000})} = 1000(1 - e^{-500/1000})$.

In 2007 the loss random variable is $Y = 1.05X$. The exponential distribution is a scale distribution, which means that a constant multiple is also exponential with a scaled mean. Therefore, Y has an exponential distribution with mean 1050.
The expected cost per payment in 2007 (same deductible and policy limit as 2006)
$\frac{E[Y\wedge 600]-E[Y\wedge 100]}{1-F_Y(100)} = \frac{1050(1-e^{-600/1050})-1050(1-e^{-100/1050})}{1-(1-e^{-100/1050})} = 1050(1 - e^{-500/1050})$.

The ratio of expected cost per payment in 2007 to that of 2006 is
$\frac{1050(1-e^{-500/1050})}{1000(1-e^{-500/1000})} = 1.011$, an increase of 1.1% from 2006 to 2007.

An alternative way to find the expected cost per payment in 2007 is as follows.
If $Y = (1 + r)X$ and a maximum covered loss of u is applied to Y, then it is possible to formulate the expected cost per loss for Y in terms of X: $E[Y \wedge u] = (1 + r)E[X \wedge \frac{u}{1+r}]$.
Also, the distribution function of Y is $F_Y(y) = F_X(\frac{y}{1+r})$.
If there is a policy deductible of d and a maximum covered loss of u applied to Y, then the expected cost per loss is
$E[Y \wedge u] - E[Y \wedge d] = (1 + r)(E[X \wedge \frac{u}{1+r}] - E[X \wedge \frac{d}{1+r}])$.
The expected cost per payment is $\frac{E[Y\wedge u]-E[Y\wedge d]}{1-F_Y(d)} = \frac{(1+r)(E[X\wedge\frac{u}{1+r}]-E[X\wedge\frac{d}{1+r}])}{1-F_X(\frac{d}{1+r})}$.
Applying this to exponential random variable X with $r = .05$, $\theta = 1000$, $d = 100$ and $u = 600$, we get an expected cost per payment in 2007 of
$\frac{(1.05)(E[X\wedge\frac{600}{1.05}]-E[X\wedge\frac{100}{1.05}])}{1-F_X(\frac{100}{1.05})} = \frac{(1.05)[1000(1-e^{-600/1050})-1000(1-e^{-100/1050})]}{1-(1-e^{-100/1050})}$.
This reduces to the same expression as the first approach.