EXAM P QUESTIONS OF THE WEEK

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Week of March 27/06

An actuarial trainee working on loss distributions encounters a version of the Pareto distribution. The student reads a discussion of the distribution and sees that the density of X is
$f(x) = \frac{\alpha\theta^\alpha}{x^{\alpha+1}}$ on the region $X > \theta$, where α and θ must both be > 0,
and the mean is $\frac{\alpha\theta}{\alpha-1}$ if $\alpha > 1$.
The student is analyzing loss data that is assumed to follow a Pareto distribution, but the values of α and θ are not specified, although it is known that $\theta < 200$. The data shows that the average loss for all losses is 180, and the average loss for all losses that are above 200 is 300.
Find the median of the loss distribution.

The solution can be found below.

Week of March 27/06 - Solution

The distribution function will be $F(y) = \int_\theta^y f(x)\,dx = \int_\theta^y \frac{\alpha\theta^\alpha}{x^{\alpha+1}}\,dx = 1 - \frac{\theta^\alpha}{y^\alpha}$.
The median m occurs where $F(m) = \frac{1}{2}$. If α and θ were known, we could find the median.

The average loss for all losses is $\frac{\alpha\theta}{\alpha-1} = 180$, but both θ and α are not known.

The conditional distribution of loss amount x given that $X > 200$ is
$f(x|X > 200) = \frac{f(x)}{P(X>200)} = \frac{\alpha\theta^\alpha}{x^{\alpha+1}} \Big/ \frac{\theta^\alpha}{200^\alpha} = \frac{\alpha 200^\alpha}{x^{\alpha+1}}$.
This is a Pareto density with $\theta = 200$, so the mean is $\frac{200\alpha}{\alpha-1}$. We are given that this mean is 300, so $\frac{200\alpha}{\alpha-1} = 300$, and therefore $\alpha = 3$.

Then, from $\frac{\alpha\theta}{\alpha-1} = 180$, we get $\frac{3\theta}{2} = 180$, so that $\theta = 120$.

The median m satisfies the relation $\frac{1}{2} = F(m) = 1 - \frac{\theta^\alpha}{m^\alpha} = 1 - (\frac{120}{m})^3$, so that $m = 151.2$.