

# EXAM FM QUESTIONS OF THE WEEK

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## Week of March 6/06

An investment fund guarantees that the value of the fund never decreases as time goes on. At the start of a year, 100 is invested into the fund. Halfway through the year, an additional 10 is deposited into the fund. The value of the fund at the end of the year is 120. Find the maximum and minimum time-weighted yields that could be reported for this investment for the year.

**The solution can be found below.**

## **Week of March 6/06 - Solution**

The balance at time 0 is 100, the balance at time  $\frac{1}{2}$  before the deposit is  $A$ , the balance at time  $\frac{1}{2}$  just after the deposit is  $A + 10$ , and the balance at time 1 is 120. The time weighted return is  $i$ , where

$$1 + i = \frac{A}{100} \cdot \frac{120}{A+10} = (1.2)\left(\frac{A}{A+10}\right).$$

Since the fund value never decreases, we know that  $100 \leq A$  and  $A + 10 \leq 120$ , so that  $100 \leq A \leq 110$ .

The minimum time-weighted yield will occur where  $\frac{A}{A+10}$  is a minimum, and the maximum time-weighted yield will occur where  $\frac{A}{A+10}$  is a maximum. The derivative with respect to  $A$  of  $\frac{A}{A+10}$  is  $\frac{10}{(A+10)^2}$ , which is  $> 0$  for  $100 \leq A \leq 110$ . Therefore, the minimum time-weighted yield occurs if  $A = 100$ , and this minimum yield is  $(1.2)\left(\frac{100}{110}\right) - 1 = .0909$ , and the maximum time-weighted yield occurs if  $A = 110$ , and this maximum yield is  $(1.2)\left(\frac{110}{120}\right) - 1 = .1000$ .