EXAM M QUESTIONS OF THE WEEK

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Week of March 6/06

A frailty model has a base age-at-death distribution that follows DeMoivre's Law with $\omega = 100$, and associated hazard rate function $a(x)$. The conditional hazard rate for the age-at-death random variable X for an individual with parameter λ is $h_{X|\lambda}(x|\lambda) = \lambda a(x)$. For a new-born individual in the frailty model group, the value of λ is uniformly distributed between .8 and 1.5. Find the probability that a randomly selected new-born from the frailty group will survive to at least age 80.

The solution can be found below.

Week of March 6/06 - Solution

The base hazard rate function is $a(x) = \frac{\lambda}{100-x}$, because DeMoivre's Law with upper age limit ω has hazard rate function (force of mortality) $\frac{1}{\omega - x}$ at age x.

The conditional survival probability to age 80 for an individual with parameter λ is
$S(80|\lambda) = e^{-\int_0^{80} \lambda\, h(x)\, dx} = [S(80)]^{\lambda} = (\frac{100-80}{100})^{\lambda} = (.2)^{\lambda}$.

The pdf of the parameter λ is $f_{\Lambda}(\lambda) = \frac{1}{.7}$ (uniform distribution on $(.8, 1.5)$).

The survival probability to age 80 for a randomly chosen individual is
$S(80) = \int_{.8}^{1.5} S(80|\lambda)\, f_{\Lambda}(\lambda)\, d\lambda = \int_{.8}^{1.5} (.2)^{\lambda} \cdot \frac{1}{.7}\, d\lambda = \frac{(.2)^{1.5} - (.2)^{.8}}{(ln\, .2)(.7)} = .166$.