EXAM FM QUESTIONS OF THE WEEK

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Week of May 15/06

A 5-year continuous annuity has the following schedule of payments:

1 per year in the first year, 2 per year in the 2nd year, 3 per year in the 3rd year,

2 per year in the 4th year, and 1 per year in the 5th year.

The force of interest is .05 . Find the present value of the annuity at the start of the first year.

The solution can be found below.

Week of May 15/06 - Solution

We can find the pv of each year's payments separately:

$$\overline{a}_{\overline{1}|} + 2v\overline{a}_{\overline{1}|} + 3v^2\overline{a}_{\overline{1}|} + 2v^3\overline{a}_{\overline{1}|} + v^4\overline{a}_{\overline{1}|} = (1 + 2v + 3v^2 + 2v^3 + v^4)\,\overline{a}_{\overline{1}|}$$

$$= (1 + 2e^{-\delta} + 3e^{-2\delta} + 2e^{-3\delta} + e^{-4\delta})(\tfrac{1-e^{-\delta}}{\delta}) = 7.96 \ .$$

Alternatively, the pv is equal to $(1 + v + v^2)\,\overline{a}_{\overline{3}|} = 7.96$.

The reason this is true, is that

1st annuity: $\overline{a}_{\overline{3}|}$ pays 1 from time 0 to time 3,

2nd annuity: $v\overline{a}_{\overline{3}|}$ pays 1 from time 1 to time 4, and

3rd annuity: $v^2\overline{a}_{\overline{3}|}$ pays 1 from time 2 to time 5 .

From time 0 to time 1 only the 1st annuity is paying, for a total amount of 1.

From time 1 to time 2, the 1st and 2nd annuities are both paying, for a total amount of 2.

From time 2 to time 3, all three annuities are paying, for a total amount of 3.

From time 3 to time 4, the 2nd and 3rd annuities are paying, for a total of 2.

From time 4 to time 5, only the 3rd annuity is paying, for a total of 1.