EXAM M QUESTIONS OF THE WEEK

S. Broverman, 2006

Week of May 1/06

A two-state homogeneous Markov chain is being used to model the transitions between days with rain (R) and without rain (N). You are given $Q^{(R,R)} = .5$, $Q^{(N,N)} = .75$.

If it is raining today, find the expected number of non-rainy days until the next rainy day.

The solution can be found below.

Week of May 1/06 - Solution

Let c be the expected number of non-rainy days until the next rainy day if it is raining today, and let d be the expected number of future non-rainy days until the next rainy day if it is not raining today. Conditioning over the weather tomorrow, we have

$c = (0) \cdot Q^{(R,R)} + (1 + d) \cdot Q^{(R,N)} = \frac{1}{2}d + \frac{1}{2}$.

We get an equation for d in the same way

$d = (0)Q^{(N,R)} + (1 + d)Q^{(N,N)} = \frac{3}{4}d + \frac{3}{4}$. From this equation we get $d = 3$.

Then from the earlier equation we get $c = 2$.