EXAM P QUESTIONS OF THE WEEK

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Week of May 1/06

X has a uniform distribution on the interval $(0,1)$.

The random variable Y is defined by $Y = X^{-k}$, where $k > 0$.

Find the mean of Y, if it is finite.

The solution can be found below.

Week of May 1/06 - Solution

$X = Y^{-1/k} = h(Y)$.

According to the method by which we find the density of a transformed random variable, the pdf of Y is $g(y) = f(h(y)) \cdot |h'(y)|$, where f is the pdf of X.

Since X is uniform on $(0,1)$, we know that $f(x) = 1$.
Therefore, $g(y) = \left| -\frac{y^{-(k+1)/k}}{k} \right| = \frac{y^{-(k+1)/k}}{k}$. Since $y = x^{-k}$, it follows that $y > 1$, since $0 < x < 1$.

The mean of Y will be $\int_1^{\infty} y \cdot g(y)\, dy = \int_1^{\infty} y \cdot \frac{y^{-(k+1)/k}}{k}\, dy = \int_1^{\infty} \frac{y^{-1/k}}{k}\, dy$
$= \frac{y^{(k-1)/k}}{k-1} \Big|_{y=1}^{y=\infty}$.
This will be ∞ if $k \geq 1$.

If $k < 1$, then $E[Y] = -\frac{1}{k-1} = \frac{1}{1-k}$.