EXAM C QUESTIONS OF THE WEEK

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Week of May 8/06

You are given the following 4-point data set

x :	0	1	2	3
y :	0	0	6	24

Find $f'(0)$ for the natural cubic spline $f(x)$.

Solution can be found below.

Week of May 8/06 - Solution

$m_0 = m_3 = 0$

$h_0 m_0 + 2(h_0 + h_1)m_1 + h_1 m_2 = 6[\frac{y_2 - y_1}{h_1} - \frac{y_1 - y_0}{h_0}]$

$\rightarrow 4m_1 + m_2 = 6[\frac{6-0}{1} - \frac{0-0}{1}] = 36$

$h_1 m_1 + 2(h_1 + h_2)m_2 + h_2 m_3 = 6[\frac{y_3 - y_2}{h_2} - \frac{y_2 - y_1}{h_1}]$

$\rightarrow m_1 + 4m_2 = 6[\frac{24-6}{1} - \frac{6-0}{1}] = 72$

$\rightarrow m_1 = \frac{24}{5}$, $m_2 = \frac{84}{5}$.

Then $a_0 = y_0 = 0$, $b_0 = \frac{y_1 - y_0}{h_0} - \frac{h_0(2m_0 + m_1)}{6} = \frac{0-0}{1} - \frac{(1)(0+4.8)}{6} = -.8$,

$c_0 = \frac{m_0}{2} = 0$, $d_0 = \frac{m_1 - m_0}{6h_0} = \frac{4.8}{6} = .8$.

$f_0(x) = 0 - .8(x - 0) + .8(x - 0)^3 = -.8x + .8x^3$.

$f_0'(0) = b_0 = -.8$.