EXAM P QUESTIONS OF THE WEEK

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Week of May 8/06

The marginal distributions of X and Y are both normal with mean 0, but X has a variance of 1, and Y has a variance of 4.

X and Y have a bivariate normal distribution with the following joint pdf:

$$f(x,y) = \frac{.3125}{\pi} \cdot e^{-.78125(x^2-.6xy+.25y^2)} \, .$$

Find the coefficient of correlation between $X+Y$ and $X-Y$.

The solution can be found below.

Week of May 8/06 - Solution

If X and Y have a bivariate normal distribution for which
X has mean μ_X and standard deviation σ_X, and
Y has mean μ_Y and standard deviation σ_Y, and
the coefficient of correlation between X and Y is ρ, then
the general bivariate normal joint pdf is
$f(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \cdot exp[-\frac{1}{2(1-\rho^2)} \cdot [(\frac{x-\mu_X}{\sigma_X})^2 + (\frac{y-\mu_Y}{\sigma_Y})^2 - 2\rho(\frac{x-\mu_X}{\sigma_X})(\frac{y-\mu_Y}{\sigma_Y})]$.

We are given that $f(x,y) = \frac{.3125}{\pi} \cdot e^{-.78125(x^2-.6xy+.25y^2)}$.
From the general form of the joint pdf, we see that $\frac{2\rho}{\sigma_X\sigma_Y} = .6$, so that $\rho = .6$.

The covariance between $X+Y$ and $X-Y$ is

$Cov(X+Y, X-Y) = Cov(X,X) + Cov(X,-Y) + Cov(Y,X) + Cov(Y,-Y)$
$= Var(X) - Cov(X,Y) + Cov(Y,X) - Var(Y) = Var(X) - Var(Y) = 1 - 4 = -3$.
The coefficient of correlation between $X+Y$ and $X-Y$ is $\frac{Cov(X+Y,X-Y)}{\sqrt{Var(X+Y)\cdot Var(X-Y)}}$.
$Var(X+Y) = Var(X) + Var(Y) + 2\rho\sqrt{Var(X)\cdot Var(Y)} = 1 + 4 + 2(.6)\sqrt{(1)(4)} = 7.4$
and
$Var(X-Y) = Var(X) + Var(Y) - 2\rho\sqrt{Var(X)\cdot Var(Y)} = 1 + 4 - 2(.6)\sqrt{(1)(4)} = 2.$

The coefficient of correlation between $X+Y$ and $X-Y$ is
$\frac{Cov(X+Y,X-Y)}{\sqrt{Var(X+Y)\cdot Var(X-Y)}} = \frac{-3}{\sqrt{(7.4)(2.6)}} = -.684$.