EXAM MFE QUESTIONS OF THE WEEK

S. Broverman, 2007

Week of April 16/07

A non-dividend paying stock has annual volatility of $\sigma = .2$ and a current price of 100.
Risk free interest is a continuously compounded rate of $r = .05$.
The CRR binomial model with n branches to time T has $u = e^{\sigma\sqrt{T/n}}$ and $d = \frac{1}{u}$.
The CRR model is applied with $T = 1$ and $n = 8$.

Based on the CRR binomial model, find the value at time 0 of a 1 year European call option
with a strike price of 120.

The solution can be found below.

Week of April 16/07 - Solution

The option is in the money only for the three largest stock prices at time 1.

The three largest stock values are

$100(1.073271)^8 = 176.0759$ (8 steps up) ,

$100(1.073271)^7 \cdot \frac{1}{1.073271} = 152.8468$ (7 steps up, 1 step down, net 6 steps up)

$100(1.073271)^6 \cdot \frac{1}{(1.073271)^2} = 132.6898$ (6 steps up, 2 steps down, net 4 steps up) .

The one step up risk neutral probability is $p^* = \frac{e^{r/n}-d}{u-d} = \frac{e^{.05/8}-\frac{1}{1.073271}}{1.073271-\frac{1}{1.073271}} = .526625.$

The risk neutral probability of a stock price of 176.0759 is $(p^*)^8 = .005916.$

The risk neutral probability of a stock price of 152.8468 is $8(p^*)^7(1-p^*) = .042541.$

The risk neutral probability of a stock price of 132.6898 is $\binom{8}{2}(p^*)^6(1-p^*)^2 = .133837.$

Option values at time 1 with risk neutral probabilities are

56.0759 , prob. .005916   or   32.8648 , prob. .042541   or   12.6898 , prob. .133837 .

The risk neutral expected value of the option at time 0 is

$e^{-.05}[(56.0759)(.005916) + (32.8648)(.042541) + (12.6898)(.133837)] = 3.261$.