EXAM MFE QUESTIONS OF THE WEEK

S. Broverman, 2007

Week of April 30/07

Today's exchange rate between Canadian and US currency is $1US = $1.2000CDN .
A binomial model for exchange rates has $u = 1.1$ and $d = .9$ each year for the next 2 years.
The annual effective risk free rate of interest in the US is 6% each year for the next 2 years, and in
Canada it is 4%. Construct the 2 year (2 step binomial tree for the $US in terms of $CDN and show that
the risk neutral probability p^* of an up-step at any node in the tree is .405660. Verify that the exchange
rate at time 0 is consistent with the risk-neutral expected present value of the three possible
outcomes at time 2.

The solution can be found below.

Week of April 30/07 - Solution

The tree is (the brackets identify node numbers)

Time 0	Time 1	Time 2
		1.4520 (4)
	1.3200 (2)	
1.2000 (1)		1.1880 (5)
	1.0800 (3)	
		.9720 (6)

$$p^* = \frac{e^{r-r_f} - d}{u - d} = \frac{\frac{1.04}{1.06} - .9}{1.1 - .9} = .405660 \ .$$

The risk neutral probabilities at time 2 are:

node (4): $(p^*)^2 = .164560$, node (5): $2(p^*)(1 - p^*) = .482200$,

node (6): $(1 - p^*)^2 = .353240$.

Expected present value at time 0 is

$$\frac{1}{(1.04)^2}[(1.4520)(.164560) + (1.1880)(.482200) + (.9720)(.353240)] = 1.0680 \ .$$

This should be equal to the value in \$CDN of $e^{-2r_f} = \frac{1}{(1.06)^2}$ \$US at time 0.

This value is $\frac{1}{(1.06)^2} \times 1.20 = 1.0680$.