

EXAM MFE QUESTIONS OF THE WEEK

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Week of April 9/07

A non-dividend paying stock has annual volatility of $\sigma = .2$ and a current price of 100.

Risk free interest is a continuously compounded rate of $r = .05$.

The CRR binomial model with n branches to time T has $u = e^{\sigma\sqrt{T/n}}$ and $d = \frac{1}{u}$.

The CRR model is applied with $T = 1$ and $n = 8$.

There are 9 possible values of the stock price at time 1 under the CRR model. Calculate the 3 largest possible stock price values at time 1 and their risk neutral probabilities of occurring (as of time 0). Find the risk-neutral probability of the stock closing over 120 at the end of the year.

The solution can be found below.

Week of April 9/07 - Solution

$$u = e^{\sigma\sqrt{T/n}} = e^{(.2)\sqrt{1/8}} = 1.073271 .$$

The three largest stock values are

$$100(1.073271)^8 = 176.0759 \text{ (8 steps up) ,}$$

$$100(1.073271)^7 \cdot \frac{1}{1.073271} = 152.8468 \text{ (7 steps up, 1 step down, net 6 steps up) ,}$$

$$100(1.073271)^6 \cdot \frac{1}{(1.073271)^2} = 132.6898 \text{ (6 steps up, 2 steps down, net 4 steps up) .}$$

$$\text{The one step up risk neutral probability is } p^* = \frac{e^{r/n} - d}{u - d} = \frac{e^{.05/8} - \frac{1}{1.073271}}{1.073271 - \frac{1}{1.073271}} = .526625.$$

$$\text{The risk neutral probability of a stock price of 176.0759 is } (p^*)^8 = .005916.$$

$$\text{The risk neutral probability of a stock price of 152.8468 is } 8(p^*)^7(1 - p^*) = .042541.$$

$$\text{The risk neutral probability of a stock price of 132.6898 is } \binom{8}{2}(p^*)^6(1 - p^*)^2 = .133837.$$

$$\text{The 4th largest value of the stock price at time 1 is } 100(1.073271)^2 = 115.19.$$

The risk neutral probability of the stock closing above 120 is the probability that the stock closes at one of the three largest closing prices. This probability is

$$.005916 + .042541 + .133837 = .182294 .$$