EXAM MFE QUESTIONS OF THE WEEK

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Week of January 15/07

A stock has a price at time 0 of $100. The force of interest is 10%.
Suppose that an investor is willing to buy a prepaid forward contract at a prepaid forward price of $105 on a one year prepaid forward contract. Show how to make an arbitrage gain under these circumstances.

The solution can be found below.

Week of January 15/07 - Solution

A simple, but general principle in finance is to buy low and sell high in order to make a gain. To create an arbitrage gain, we want to buy low and sell high. If we can "buy" for 0 and "sell" for more than 0, then we have made an arbitrage gain. Since the current price of the stock is 100, if someone is offering a prepaid forward at a price that is not 100 we can take advantage of the situation in one of two possible ways, depending upon what the prepaid forward price is.
If the prepaid forward price being offered is more than 100, we sell the prepaid forward and buy the stock, and vice versa if the prepaid forward price that an investor is willing to accept is less than 100.

We sell to the investor the prepaid one-year forward contract on the stock (this is the "sell high" part of the arrangement). This means that the investor pays us $105 right now (time 0), and we agree to deliver a share of stock to the investor one year from now. We use $100 of the amount we receive to buy the stock right now (this is the "buying low" part of the arrangement), and invest the remaining $5 at the force of interest of 10%. In one year, we have the stock to deliver to the investor to complete the forward transaction, and we have cash of $5e^{.1} = 5.53$. This positive gain was obtained with a net investment of 0 at time 0.