EXAM MLC QUESTIONS OF THE WEEK

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Week of February 4/08

Which of the following relationships are correct?

I. $A_{\overline{x:\overline{n}|}} = 1 - d \cdot \ddot{a}_{\overline{x:\overline{n}|}}$ ($A_{\overline{x:\overline{n}|}}$ is the single benefit premium for a payment on the later of the end of year of (x)'s death and n years, and $\ddot{a}_{\overline{x:\overline{n}|}}$ is an annuity due which continues for at least n years or as long as (x) survives, whichever is later).

II.
$$n|A_x = 1 - d \cdot n|\ddot{a}_x$$

III.
$$(IA)_x = \ddot{a}_x - d \cdot (I\ddot{a}_x)$$

The solution can be found below.

Week of February 4/08 - Solution

I. Correct.

$$\begin{split} &A_{\overline{x:\overline{n}|}} = A_x + v^n - A_{x:\overline{n}|} = 1 - d \cdot \ddot{a}_x + (1 - d \cdot \ddot{a}_{\overline{n}|}) - (1 - d \cdot \ddot{a}_{x:\overline{n}|}) \\ &= 1 - d \cdot (\ddot{a}_x + \ddot{a}_{\overline{n}|} - \ddot{a}_{x:\overline{n}|}) = 1 - d \cdot \ddot{a}_{\overline{x:\overline{n}|}} \end{split}$$

II. Incorrect.

$$\begin{split} &_{n|}A_{x} = A_{x} - A_{\frac{1}{x:\overline{n}|}} = A_{x} - (A_{x:\overline{n}|} - A_{\frac{1}{x:\overline{n}|}}) \\ &= (1 - d \cdot \ddot{a}_{x}) - (1 - d \cdot \ddot{a}_{x:\overline{n}|}) + A_{\frac{1}{x:\overline{n}|}} = A_{\frac{1}{x:\overline{n}|}} - d \cdot (\ddot{a}_{x} - \ddot{a}_{x:\overline{n}|}) \\ &= A_{\frac{1}{x:\overline{n}|}} - d \cdot {}_{n|} \ddot{a}_{x} \neq 1 - d \cdot {}_{n|} \ddot{a}_{x} \ \ \text{(unless} \ \ A_{\frac{1}{x:\overline{n}|}} = v^{n} \, {}_{n} p_{x} = 1). \end{split}$$

III. Correct.

$$\begin{split} &(IA)_{x} = A_{x} + {}_{1|}A_{x} + {}_{2|}A_{x} + \cdots = \sum\limits_{k=0}^{\infty}{}_{k|}A_{x} = \sum\limits_{k=0}^{\infty}{}^{v^{k}}{}_{k}p_{x} \cdot A_{x+k} \\ &= \sum\limits_{k=0}^{\infty}{}^{v^{k}}{}_{k}p_{x} \cdot (1 - d \cdot \ddot{a}_{x+k}) = \sum\limits_{k=0}^{\infty}{}^{v^{k}}{}_{k}p_{x} - d \cdot \sum\limits_{k=0}^{\infty}{}^{v^{k}}{}_{k}p_{x} \cdot \ddot{a}_{x+k} \\ &= \ddot{a}_{x} - d \cdot \sum\limits_{k=0}^{\infty}{}_{k|} \ddot{a}_{x} = \ddot{a}_{x} - d \cdot (\ddot{a}_{x} + {}_{1|}\ddot{a}_{x} + {}_{2|}\ddot{a}_{x} + \cdots) = \ddot{a}_{x} - d \cdot (I\ddot{a}_{x}) \end{split}$$