EXAM MLC QUESTIONS OF THE WEEK

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Week of January 22/07

You are given that $\mu_{20}(t) = .02$ for $t \geq 0$ and $\delta = .1$.
Let Z denote the present value random variable for a whole life insurance issued to (20) with death benefit 1 payable at the moment of death. Find $E[Z]$ – (median of Z).

The solution can be found below.

Week of January 22/07 - Solution

With constant force at all ages above 20, we have $\omega = \infty$, and ${}_tp_{20} = e^{-.02t}$.

$E[Z] = \overline{A}_{20} = \int_0^{\infty} e^{-\delta t}\, {}_tp_{20}\, \mu_{20}(t)\, dt = \int_0^{\infty} e^{-.1t} e^{-.02t}(.02)\, dt = \frac{.02}{.12} = \frac{1}{6}$.

The median of Z, say m, must satisfy $P[Z \leq m] = .5$. We use the following equivalences to translate this probability into one involving $T(20)$.

$Z \leq m \;\Leftrightarrow\; e^{-.1T} \leq m \;\Leftrightarrow\; .1T \geq -ln(m) \;\Leftrightarrow\; T \geq -10\, ln(m)$

Then, $.5 = P[Z \leq m] = P[T \geq -10\, ln(m)]$. But $P[T \geq a] = {}_ap_{20} = e^{-.02a}$.

Therefore, $.5 = P[T \geq -10\, ln(m)] = e^{-.02\cdot[-10\, ln\,(m)]} = m^{.2} \rightarrow m = (.5)^5 = .03125 = \frac{1}{32}$.

Then $E[Z]$ – (median of Z) $= \frac{1}{6} - \frac{1}{32} = \frac{13}{96}$.