## EXAM MLC QUESTIONS OF THE WEEK

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## Week of January 7/08

Actuary Smith has mortality models for smokers and non-smokers. According to Actuary Smith's models, non-smokers have constant force of mortality  $\mu^{S}$  at all ages and smokers have constant force of mortality  $\mu^{NS}$  at all ages.

Actuary Jones has mortality models for smokers and non-smokers. According to Actuary Jones's models, non-smoker mortality follows DeMoivre's Law with upper age limit  $\omega$ , and smokers have a force of mortality which is constant multiple *c* of that of non-smokers.

At all ages, the ratio  $\frac{\hat{e}_x^{NS}}{e_x^{oS}}$  is the same for the models of Smith and Jones.

Find an expression for c in terms of Smith's  $\mu^{NS}$  and  $\mu^{S}$ .

A)  $\frac{2\mu^{NS}}{\mu^{S}}$  B)  $\frac{2\mu^{S}}{\mu^{NS}}$  C)  $\frac{2\mu^{NS}}{\mu^{S}} - 1$  D)  $\frac{2\mu^{S}}{\mu^{NS}} - 1$  E)  $\frac{\mu^{S}}{\mu^{NS}} + 1$ 

## The solution can be found below.

## Week of January 7/08 - Solution

For Smith's model,  $\hat{e}_x^{NS} = \frac{1}{\mu^{NS}}$  and  $\hat{e}_x^S = \frac{1}{\mu^S}$ , so  $\frac{\hat{e}_x^{NS}}{e^{\circ}_x^S} = \frac{\mu^S}{\mu^{NS}}$ .

For Jones's model,  $\mu^{NS}(x) = \frac{1}{\omega - x}$  and  $\mu^{S}(x) = \frac{c}{\omega - x}$ , so for Jones's model, smoker mortality follows a Generalized DeMoivre Law with upper age limit  $\omega$  and parameter c. For the Jones model, complete expectation for non-smokers is  $e_x^{NS} = \frac{\omega - x}{2}$  and for smokers it is  $e_x^{S} = \frac{\omega - x}{c+1}$ . For Jones's model,  $\frac{e_x^{NS}}{e_x^{oS}} = \frac{c+1}{2}$ .

Therefore,  $\frac{c+1}{2} = \frac{\mu^S}{\mu^{NS}}$ , and  $c = \frac{2\mu^S}{\mu^{NS}} - 1$ . Answer: D