EXAM MLC QUESTIONS OF THE WEEK

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Week of March 19/07

(CAS Nov 05) For a 20-year term life insurance on (x), you are given:

- $i = 0$
- A benefit of 2 is paid if death occurs by accident.
- A benefit of 1 is paid if death occurs from all other causes.
- Benefits are paid at the moment of death.
- The force of mortality for death by accident is $\mu_{x+t}^{(1)} = \frac{t}{20}$.
- The force of mortality for all other causes of death is $\mu_{x+t}^{(2)} = \frac{t}{10}$.

Calculate the net single premium for this insurance.

A) $\frac{2}{3}(1 - e^{-20})$ B) $1 - e^{-20}$ C) $1 - e^{-30}$ D) $1 - e^{-40}$ E) $\frac{4}{3}(1 - e^{-30})$

The solution can be found below.

Week of March 19/07 - Solution

If $i=0$, the value of an n-year term insurance of 1 is

$\int_0^n v^t\,{}_tp_x\,\mu_{x+t}\,dt = \int_0^n {}_tp_x\,\mu_{x+t}\,dt = {}_nq_x$ in the continuous insurance case, and

it is also ${}_nq_x$ in the discrete term insurance case.

This problem involves a multiple decrement model, but since $i=0$, insurance values will still be probabilities.

For the 20-year term insurance for accidental death, the APV is $\int_0^{20} 2v^t\,{}_tp_x^{(\tau)}\,\mu_{x+t}^{(1)}\,dt = 2\cdot{}_{20}q_x^{(1)}$.

For the 20-year term insurance for other causes of, the APV is $\int_0^{20} v^t\,{}_tp_x^{(\tau)}\,\mu_{x+t}^{(2)}\,dt = {}_{20}q_x^{(2)}$.

We now note that $\mu_{x+t}^{(1)} = \frac{t}{20}$, $\mu_{x+t}^{(2)} = \frac{t}{10}$, and $\mu_{x+t}^{(\tau)} = \frac{3t}{20}$.

Therefore, $\mu_{x+t}^{(1)} = \frac{1}{3}\cdot\ \mu_{x+t}^{(\tau)}$, and $\mu_{x+t}^{(2)} = \frac{2}{3}\cdot\ \mu_{x+t}^{(\tau)}$, and it follows that

${}_nq_x^{(1)} = \frac{1}{3}\cdot{}_nq_x^{(\tau)}$ and ${}_nq_x^{(2)} = \frac{2}{3}\cdot{}_nq_x^{(\tau)}$.

From the combined force of decrement, we have ${}_{20}p_x^{(\tau)} = e^{-\int_0^{20}\mu_{x+t}^{(\tau)}\,dt} = e^{-\int_0^{20}.15t\,dt} = e^{-30}$.

Therefore, ${}_{20}q_x^{(1)} = \frac{1}{3}\cdot(1-e^{-30})$ and ${}_{20}q_x^{(2)} = \frac{2}{3}\cdot(1-e^{-30})$.

The term insurance single premium is $2[\frac{1}{3}\cdot(1-e^{-30})] + \frac{2}{3}\cdot(1-e^{-30}) = \frac{4}{3}\cdot(1-e^{-30})$.

Answer: E