EXAM MLC QUESTION OF THE WEEK

S. Broverman, 2008

Week of March 24/08

In a one decrement model, the decrement has a constant force $\mu_x^{(1)}(t) = a$ for all $t > 0$.
A second decrement is added to the model, and the second decrement also has a constant force that is the same as that of decrement 1: $\mu_x^{(2)}(t) = a$.
As a result of adding the second decrement, the probability $q_x^{(1)}$ is 25% lower in the new two decrement model than it was in the original two decrement model. Find the value of a.

The solution can be found below.

Week of March 24/08 - Solution

In the one decrement model, $q_x^{(1)} = q_x^{(\tau)} = 1 - e^{-a}$.

In the two decrement model, with constant force of decrement, we have

$\frac{q_x^{(1)New}}{q_x^{(\tau)New}} = \frac{\mu_x^{(1)}}{\mu_x^{(\tau)New}} = \frac{a}{a+a} = \frac{1}{2}$, so that

$q_x^{(1)New} = \frac{1}{2} \times q^{(\tau)New} = \frac{1}{2} \times (1 - e^{-2a}) = (1 - e^{-a}) \times \frac{1}{2} \times (1 + e^{-a})$

$= q_x^{(1)Old} \times \frac{1}{2} \times (1 + e^{-a})$.

Therefore, $\frac{1}{2} \times (1 + e^{-a}) = .75$ and $a = -ln\,.5 = .693$