EXAM MLC QUESTION OF THE WEEK

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Week of March 31/08

For a three decrement model,you are given the following absolute survival rates from the associated single decrement tables for age x: $p_x'^{(1)} = .8$, $p_x'^{(2)} = .6$, $p_x'^{(3)} = .4$.
(a) Assume that decrements 1 and 2 are UDD in their associated single decrement tables in each year of age, and decrement 3 is discrete. Decrement 3 occurs at time $\frac{1}{4}$ and time $\frac{3}{4}$, with one-half of the annual absolute rate occurring at time $\frac{1}{4}$ and the other half at time $\frac{3}{4}$.

Find $q_x^{(1)}$.

The solution can be found below.

Week of March 31/08 - Solution

$p_x^{(\tau)} = (.8)(.6)(.4) = .192 \rightarrow q_x^{(\tau)} = .808$.
$q_x^{(1)} = \int_0^1 {}_tp_x^{(\tau)} \cdot \mu_x^{(1)}(t)\,dt = \int_0^1 {}_tp_x'^{(1)} \cdot {}_tp_x'^{(2)} \cdot {}_tp_x'^{(3)} \cdot \mu_x^{(1)}(t)\,dt$
From UDD in single tables 1 and 2, we have
${}_tp_x'^{(1)} \cdot \mu_x^{(1)}(t) = q_x'^{(1)} = .2$ and ${}_tp_x'^{(1)} = 1 - t \cdot q_x'^{(2)} = 1 - .4t$.
Then, $q_x^{(1)} = (.2) \times \int_0^1 (1 - .4t) \cdot {}_tp_x'^{(3)}\,dt$.
From the description of ${}_tp_x'^{(3)}$, we have ${}_tp_x'^{(3)} = \begin{cases} 1 & 0 < t < .25 \\ .7 & .25 \leq t < .75 \\ .4 & .75 \leq t < 1 \end{cases}$.
Then, $q_x^{(1)} = (.2)[\int_0^{.25}(1 - .4t)\,dt + \int_{.25}^{.75}(1 - .4t)(.7)\,dt + \int_{.75}^{1}(1 - .4t)(.4)\,dt]$
$= (.2)[\frac{19}{80} + \frac{7}{25} + \frac{13}{200}] = .1165$.