EXAM MLC QUESTION OF THE WEEK

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Week of May 5/08

The Boiler Room Sales company ranks each of its sales people at the end of each month according to the sales made by that sales person. There are three rankings:
Excellent , Good , Poor.

A salesperson who had poor sales for the month is immediately fired and never again rehired. Some sales people die during the month (perhaps because of the high pressure under which they have to work). The BRS company has created a homogeneous Markov Chain model to describe transitions in a salesperson's ranking from one month to the next. The model has three states:

E - excellent sales for the month just ended,
G - good sales for the month just ended, and
P - poor sales or died in the month just ended.

The one-step transition matrix is $\begin{bmatrix} Q^{(E,E)} & Q^{(E,G)} & Q^{(E,P)} \\ Q^{(G,E)} & Q^{(G,G)} & Q^{(G,P)} \\ Q^{(P,E)} & Q^{(P,G)} & Q^{(P,P)} \end{bmatrix} = \begin{bmatrix} .2 & .6 & .2 \\ .1 & .5 & .4 \\ 0 & 0 & 1 \end{bmatrix}$.

A salesperson has just received a ranking of G.
BRS company pays a salesperson a bonus of $1000 whenever that person has two consecutive months with a ranking of E. At a monthly interest rate of 1%, find the combined actuarial present values of the bonus payments to be made at the end of the next three months to a salesperson whose current rating is G, and a salesperson whose current rating is E .

The solution can be found below.

Week of May 5/08 - Solution

The following sequences of rankings for the next three months result in two consecutive E-E ratings (including the current ranking of G):
Sequence that pay bonus at time 2: $G-E-E$, prob. $(.1)(.2)=.02$
Sequences that pay bonus at time 3:
$G-G-E-E$, prob. $(.5)(.1)(.2)=.01$, $G-E-E-E$, prob. $(.1)(.2)(.2)=.004$,
total prob. of bonus payment at time 3 is .014.
Actuarial present value is $1000[.04v^2+.014v^3]=52.80$.

The following sequences of rankings for the next three months result in two consecutive E-E ratings (including the current ranking of E):
Sequence that pays bonus at time 1: $E-E$, prob. .2
Sequence that pay bonus at time 2: $E-E-E$, prob. $(.2)(.2)=.04$
Sequences that pay bonus at time 3:
$E-G-E-E$, prob. $(.6)(.1)(.2)=.012$, $E-E-E-E$, prob. $(.2)(.2)(.2)=.008$,
total prob. of bonus payment at time 3 is .02.
Actuarial present value is $1000[.2v+.04v^2+.02v^3]=256.64$.

Total APV is 309.44 .