EXAM MLC QUESTIONS OF THE WEEK

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A gambler is playing a game in which he either wins 1 dollar or loses 1 dollar on each play of the game. The probability of winning 1 on any play of the game is .4. If he ever reaches 0 dollars, he stops gambling and stays at 0 dollars. Suppose that he currently has 2 dollars. Find the minimum number of plays n so that the probability of reaching 0 by time n is at least .5.

The solution can be found below.

Week of October 29/07 - Solution

The general one-step transition probability matrix for this transition process has the form

State →	0	1	2	3	4	5	6	7	⋯
State ↓									
0	1	0	0	0	0	0	0	0	⋯
1	.6	0	.4	0	0	0	0	0	⋯
2	0	.6	0	.4	0	0	0	0	⋯
3	0	0	.6	0	.4	0	0	0	⋯
4	0	0	0	.6	0	.4	0	0	⋯
5	0	0	0	0	.6	0	.4	0	⋯
6	0	0	0	0	0	.6	0	.4	⋯

The process starts in state 2, so after one transition, row 2 of the two step transition matrix is the row resulting from multiplying the row vector $[\,0 \;\; .6 \;\; 0 \;\; .4 \;\; 0 \;\; \cdots\,]$ by the one-step transition matrix above. This results in the row vector.

State →	0	1	2	3	4	5	6	7	⋯
State 2	.36	0	.48	0	.16	0	0	0	⋯

These are the two step transition probabilities from initial state 2. We see that the probability of reaching 0 by time 2 is .36 . We multiply this row vector by the one-step transition matrix to get the following row vector.

State →	0	1	2	3	4	5	6	7	⋯
State 2	.36	.288	0	.288	0	.064	0	0	⋯

These are the three step transition probabilities from initial state 2. We see that the probability of reaching 0 by time 3 is .36 . We multiply this row vector by the one-step transition matrix to get the following row vector.

State →	0	1	2	3	4	5	6	7	⋯
State 2	.5328	0	.288	0	.1536	0	.0256	0	⋯

These are the four step transition probabilities from initial state 2. We see that the probability of reaching 0 by time 4 is .5328 which is greater than .5 . Time $n = 4$ is the first n for which the probability of reaching state 0 by time n is at least .5.