EXAM MLC QUESTIONS OF THE WEEK

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Week of September 10/07

It is January 1, 2008 and (x) and (y) are two independent lives.
(x) has a constant force of mortality of .03 until his death,
and (y) has a constant force of mortality of .02 until his death.
Find the probability that (y) dies either in the same calendar year as (x) dies,
or in the calendar year following the calendar year of (x)'s death.

The solution can be found below.

Week of September 10/07 - Solution

For (x) we have $p_{x+k} = e^{-.03}$ for all k, and
for (y) we have $p_{y+k} = e^{-.02}$ for all k.

Therefore, ${}_{k|}q_x = {}_kp_x \cdot q_{x+k} = e^{-.03k} \cdot (1 - e^{-.03})$
and ${}_{k|}q_y = {}_kp_y \cdot q_{y+k} = e^{-.02k} \cdot (1 - e^{-.02})$.

The probability that (x) and (y) die in the same calendar year is

$$P[K(x) = K(y)] = \sum_{k=0}^{\infty} P[(K(x) = k) \cap (K(y) = k)]$$

$$= \sum_{k=0}^{\infty} P[K(x) = k] \cdot P[K(y) = k] \text{ (because of independence of } K(x) \text{ and } K(y))$$

$$= \sum_{k=0}^{\infty} {}_{k|}q_x \cdot {}_{k|}q_y = \sum_{k=0}^{\infty} e^{-.03k} \cdot (1 - e^{-.03}) \cdot e^{-.02k} \cdot (1 - e^{-.02})$$

$$= (1 - e^{-.03})(1 - e^{-.02}) \cdot \sum_{k=0}^{\infty} e^{-.05k} = (1 - e^{-.03})(1 - e^{-.02}) \cdot \sum_{k=0}^{\infty} (e^{-.05})^k$$

$$= \frac{(1-e^{-.03})(1-e^{-.02})}{1-e^{-.05}} .$$

The probability that (y) dies in the calendar year following the calendar year of (x)'s death is

$$P[K(x) + 1 = K(y)] = \sum_{k=0}^{\infty} P[(K(x) = k) \cap (K(y) = k+1)] = \sum_{k=0}^{\infty} {}_{k|}q_x \cdot {}_{k+1|}q_y$$

$$= \sum_{k=0}^{\infty} e^{-.03k} \cdot (1 - e^{-.03}) \cdot e^{-.02(k+1)} \cdot (1 - e^{-.02})$$

$$= e^{-.02} \cdot P[K(x) = K(y)] = e^{-.02} \cdot \frac{(1-e^{-.03})(1-e^{-.02})}{1-e^{-.05}} .$$

The total probability is $(1 + e^{-.02}) \cdot \frac{(1-e^{-.03})(1-e^{-.02})}{1-e^{-.05}} = \frac{(1-e^{-.03})(1-e^{-.04})}{1-e^{-.05}} = .02376$.