EXAM FM QUESTIONS OF THE WEEK

S. Broverman, 2006

Week of October 16/06

A bond has annual coupons for $n > 1$ years at rate $r > 0$, face and redemption amount F, and yield to maturity j. The duration is D. Find each of the following limits:

(i) $\lim\limits_{r \to \infty} D$

(ii) $\lim\limits_{n \to \infty} D$

(iii) $\lim\limits_{j \to \infty} D$

The solution can be found below.

Week of October 16/06 - Solution

$$D = [Fr \cdot (Ia)_{\overline{n}|j} + nFv^n]/[r \cdot a_{\overline{n}|j} + Fv^n] = \frac{Fr(v+2v^2+\cdots+nv^n)+nFv^n}{Fr(v+v^2+\cdots+v^n)+Fv^n} \ .$$

(i) $D = [(Ia)_{\overline{n}|j} + n \cdot \frac{1}{r}v^n]/[a_{\overline{n}|j} + \frac{1}{r}v^n] \rightarrow (Ia)_{\overline{n}|j}/[a_{\overline{n}|j}$ as $r{\rightarrow}\infty$.

(ii) $D = [Fr \cdot (Ia)_{\overline{n}|j} + nFv^n]/[Fr \cdot a_{\overline{n}|j} + Fv^n] \rightarrow [Fr \cdot (Ia)_{\overline{\infty}|j}]/[Fr \cdot a_{\overline{\infty}|j}]$

$= (\frac{1}{i} + \frac{1}{i^2})/(\frac{1}{i}) = 1 + \frac{1}{i} = \frac{1}{d}$. Note that $nv^n = \frac{n}{(1+j)^n} \rightarrow 0$ as $n{\rightarrow}\infty$

follows from l"Hospital's calculus limit rule.

(iii) $D = \frac{Fr(v+2v^2+\cdots+nv^n)+nFv^n}{Fr(v+v^2+\cdots+v^n)+Fv^n}$

$= \frac{Fr(1+2v+\cdots+nv^{n-1})+nFv^{n-1}}{Fr(1+v+\cdots+v^{n-1})+Fv^{n-1}} \rightarrow \frac{r}{r} = 1$, since $v^k \rightarrow 0$ as $j{\rightarrow}\infty$ for any $k > 0$.