EXAM M QUESTIONS OF THE WEEK

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Week of October 16/06

X is a loss random variable.

An insurance policy pays nothing for losses below an ordinary deductible of 10.
The insurance pays half of the loss amount in excess of 10 up to a loss of amount 50.
The insurance pays the full loss amount for any loss in excess of 50.
Express the expected cost per loss in terms of factors of the form $E[X \wedge x]$, $F_X(x)$
and constants.

The solution can be found below.

Week of October 16/06 - Solution

The amount paid by the insurance is $Y = \begin{cases} 0 & X \leq 10 \\ \frac{1}{2}(X - 10) & 10 < X \leq 50 \\ X & X > 50 \end{cases}$.

$$\frac{1}{2}[(X \wedge 50) - (X \wedge 10)] = \begin{cases} 0 & X \leq 10 \\ \frac{1}{2}(X - 10) & 10 < X \leq 50 \\ 20 & X > 50 \end{cases}$$.

$$(X - 50)_+ = X - (X \wedge 50) = \begin{cases} 0 & X \leq 50 \\ X - 50 & X > 50 \end{cases}$$.

Therefore,

$$\frac{1}{2}[(X \wedge 50) - (X \wedge 10)] + X - (X \wedge 50) = \begin{cases} 0 & X \leq 10 \\ \frac{1}{2}(X - 10) & 10 < X \leq 50 \\ X - 30 & X > 50 \end{cases}$$.

Therefore $Y = \frac{1}{2}[(X \wedge 50) - (X \wedge 10)] + X - (X \wedge 50)$

$\quad\quad\quad$ + a payment of 30 if $X > 50$,

so that the expected cost per loss is

$$E[Y] = E[\tfrac{1}{2}[(X \wedge 50) - (X \wedge 10)] + X - (X \wedge 50)] + (30)[1 - F_X(50)] .$$