EXAM FM QUESTIONS OF THE WEEK

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Question 13 - Week of October 17

You are given the following payment schedule for a perpetuity. Interest is at an annual effective rate of 8%.

Date	Amount
1/1/06	$10
1/1/07	30
1/1/08	50
1/1/09	70
1/1/10	90
1/1/11 and each 1/1 thereafter	110

In what range is the present value of the perpetuity as of 1/1/06?

The solution can be found below.

Question 13 Solution

There is more than one way to approach this problem. One approach would be to separate the first 5 increasing payments from the level payments of 110 each continuing from 1/1/11, and find the pv of each of the two groups. A more efficient solution is illustrated by expressing the original series as a combination outlined in the following time diagram; time 0 corresponds to 1/1/06.

	1/1/06	07	08	09	10	11	12	13	
	0	1	2	3	4	5	6	7	$\cdots$
original series	10	30	50	70	90	110	110	110	$\cdots$
new series 1	110	110	110	110	110	110	110	110	$\cdots$
new series 2	-100	-80	-60	-40	-20				

Note that new series 2 has <u>negative</u> payments, so its pv will be negative.

The pv of new series 1 at 1/1/06 is $110\ddot{a}_{\overline{\infty}|.08} = 110(\frac{1}{d}) = 110(\frac{1+i}{i}) = 110(\frac{1.08}{.08}) = 1485$.

The pv of new series 2 at 1/1/06 is $-20(D\ddot{a})_{\overline{5}|.08} = -20(1+i)(Da)_{\overline{5}|.08}$.

We calculate $(Da)_{\overline{5}|.08} = \frac{5 - a_{\overline{5}|.08}}{i} = 12.591$, and the pv of new series 2 is

$-20(1.08)(12.591) = -271.97$.

The combined pv of new series 1 and new series 2 is $1485 - 272 = 1213$.