EXAM C QUESTIONS OF THE WEEK

S. Broverman, 2005

Question 15 - Week of October 31

The following random sample of size 5 is taken from the distribution of X:

1 , 3 , 4 , 7 , 10

Bootstrap approximation of the mean square error of estimators is to be based on the following 6 resamplings of size 5 from the empirical distribution:

Resample 1 : 1 , 1 , 4 , 7 , 7
Resample 2 : 3 , 4 , 4 , 7 , 10
Resample 3 : 1 , 4 , 4 , 10 , 10
Resample 4 : 3 , 3 , 3 , 4 , 10
Resample 5 : 4 , 4 , 7 , 7 , 10
Resample 6 : 1 , 7 , 7 , 10 , 10

The mean of X is estimated by the sample mean estimator, and

the 3rd moment of X is estimated by the estimator $\frac{1}{5}\sum_{i=1}^{5} X_i^3$.

Find the bootstrap approximation of the mean square error for each of those estimators using the 6 resamplings.

The solution can be found below.

Question 15 Solution

The mean of the empirical distribution is $\theta_1 = \frac{1+3+4+7+10}{5} = 5$ and the 3rd moment of the empirical distribution is $\theta_2 = \frac{1^3+3^3+4^3+7^3+10^3}{5} = 287$.

Resample	$\widehat{\theta}_1$	$(\widehat{\theta}_1 - 5)^2$	$\widehat{\theta}_2$	$(\widehat{\theta}_2 - 287)^2$
1 , 1 , 4 , 7 , 7	4	$(4-5)^2 = 1$	150.4	$(150.4 - 287)^2 = 18,660$
3 , 4 , 4 , 7 , 10	5.6	$(5.6-5)^2 = .36$	299.6	$(299.6 - 287)^2 = 158$
1 , 4 , 4 , 10 , 10	5.8	$(5.8-5)^2 = .64$	425.8	$(425.8 - 287)^2 = 19,265$
3 , 3 , 3 , 4 , 10	4.6	$(4.6-5)^2 = .16$	229	$(229 - 287)^2 = 3,364$
4 , 4 , 7 , 7 , 10	6.4	$(6.4-5)^2 = 1.96$	362.8	$(362.8 - 287)^2 = 5,746$
1 , 7 , 7 , 10 , 10	7.0	$(7.0-5)^2 = 4$	537.4	$(537.4 - 287)^2 = 62,700$

The bootstrap estimate of MSE$(\widehat{\theta}_1)$ is $\frac{1+.36+.64+.16+1.96+4}{6} = 1.353$.

The bootstrap estimate of MSE$(\widehat{\theta}_2)$ is $\frac{18,660+\cdots+62,700}{6} = 18,316$.