EXAM P QUESTION OF THE WEEK

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An insurer has n independent policies, each covering a loss with an exponential distribution with a mean of 1. Each policy has as a deductible of 1. The insurer collects a premium of .5 per policy. The insurer determines that, using the normal approximation, the probability is .01 of experiencing a net positive loss on the collection of policies. Determine n, the number of policies.

The solution can be found below.

Week of April 21/08 - Solution

If X denotes the size of the exponential loss on a policy, then the insurer's loss on an individual policy is $Y - .5$ where $Y = \begin{cases} 0 & \text{if } X \leq 1 \\ X - 1 & \text{if } X > 1 \end{cases}$.

The expected insurer's loss on a policy is $E(Y) - .5$ and the variance of the insurer's loss on a policy is $Var(Y)$.

$E(Y) = \int_1^\infty (x-1)\, e^{-x}\, dx = -xe^{-x}\Big|_{x=1}^{x=\infty} = e^{-1}$ and

$E(Y^2) = \int_1^\infty (x-1)^2\, e^{-x}\, dx = -(x^2+1)e^{-x}\Big|_{x=1}^{x=\infty} = 2e^{-1}$.

Then, $Var(Y) = E(Y^2) - [E(Y)]^2 = 2e^{-1} - e^{-2}$.

The insurer's expected loss on a single policy is $E(Y) - .5 = e^{-1} - .5$.

The total loss on the insurer's collection of policies is $S = \sum_{i=1}^{n}(Y_i - .5)$ with mean

The insurer's expected loss on n independent policies is $(e^{-1} - .5)n$ and
the variance of the insurer's loss on n independent policies is $(2e^{-1} - e^{-2})n$.

The probability of a positive loss on the insurer's collection of n policies is $P(S > 0)$.
Applying the normal approximation, this is $P(\frac{S - (e^{-1} - .5)n}{\sqrt{(2e^{-1} - e^{-2})n}} > \frac{-(e^{-1} - .5)n}{\sqrt{(2e^{-1} - e^{-2})n}})$.

In order for this probability to be .01, we must have $\frac{-(e^{-1} - .5)n}{\sqrt{(2e^{-1} - e^{-2})n}} = 2.33$.
Solving for n results in $n = 186.7$ (this would be rounded up to 187 to guarantee that the probability is at least .01).