EXAM P QUESTION OF THE WEEK

S. Broverman, 2008

Week of April 28/08

X and Y are independent loss random variables.

X has a discrete distribution that is uniformly distributed on the integers from 1 to 10, and
Y has a discrete distribution that is uniformly distributed on the integers from 1 to 20.

An insurance policy on X has a deductible of 2, and an insurance policy on Y has a deductible of 3. These two insurance policies are replaced by a single policy on $Z = X + Y$ with a single deductible d applied to the combined loss Z.

Find d so that the sum of the expected insurance payments on the two separate policies on X and Z is the same as the expected insurance payment on the policy on Z.

The solution can be found below.

Week of April 28/08 - Solution

The insurance payment for the loss X is $X - \begin{cases} 1 & \text{if } X = 1 \\ 2 & \text{if } X \geq 2 \end{cases}$,

so the expected insurance payment for the loss X is
$E(X) - [1(.1) + 2(.9)] = 5.5 - 1.9 = 3.6.$

The insurance payment for the loss Y is $Y - \begin{cases} 1 & \text{if } Y = 1 \\ 2 & \text{if } Y = 2 \\ 3 & \text{if } Y \geq 3 \end{cases}$

so the expected insurance payment for the loss Y is
$E(Y) - [1(.05) + 2(.05) + 3(.9)] = 10.5 - 2.85 = 7.65$.

The total expected payment for the two policies is $3.6 + 7.65 = 11.25$.

The total expected loss for X and Y combined is $E(Z) = E(X) + E(Y) = 5.5 + 10.5 = 16$.

With a deductible of 1 on Z the insurance payment is $Z - 1$, because $Z = X + Y$ must be ≥ 2.
The expected insurance payment would be $16 - 1 = 15$

With a deductible of 2 on Z the insurance payment is $Z - 2$, because $Z = X + Y$ must be ≥ 2.
The expected insurance payment would be $16 - 2 = 14$.

With a deductible of 3 on Z the insurance payment is $Z - \begin{cases} 2 & \text{if } Z = 2 \\ 3 & \text{if } Z \geq 3 \end{cases}$.

$P(Z = 2) = P(X = 1) \times P(Y = 1) = (.1)(.05) = .005$, and $P(Z \geq 3) = .995$.
The expected insurance payment would be $16 - [2(.005) + 3(.995)] = 13.005$.

With a deductible of 4 on Z the insurance payment is $Z - \begin{cases} 2 & \text{if } Z = 2 \\ 3 & \text{if } Z = 3 \\ 4 & \text{if } Z \geq 4 \end{cases}$

$P(Z = 3) = P(X = 1) \times P(Y = 2) + P(X = 2) \times P(Y = 1)$
$= 2(.1)(.05) = .01$, and $P(Z \geq 4) = 1 - .005 - .01 = .985$.
The expected insurance payment would be $16 - [2(.005) + 3(.01) + 4(.985)] = 12.02$.

With a deductible of 5 on Z the insurance payment is $Z \; - \begin{cases} 2 & \text{if } Z = 2 \\ 3 & \text{if } Z = 3 \\ 4 & \text{if } Z = 4 \\ 5 & \text{if } Z \geq 5 \end{cases}$

$P(Z = 4) = P(X = 1) \times P(Y = 3) + P(X = 2) \times P(Y = 2) + P(X = 3) \times P(Y = 1)$
$= 3(.1)(.05) = .015$, and $P(Z \geq 5) = 1 - .005 - .01 - .015 = .97$.
The expected insurance payment would be
$16 - [2(.005) + 3(.01) + 4(.015) + 5(.97)] = 11.05$.

We wish to find the deductible d which results in an expected insurance payment of 11.25 when applied to Z. The deductible must be between 4 and 5. The insurance payment with deductible d where $4 \leq d \leq 5$ is $Z \; - \begin{cases} 2 & \text{if } Z = 2 \\ 3 & \text{if } Z = 3 \\ 4 & \text{if } Z = 4 \\ d & \text{if } Z \geq 5 \end{cases}$ and the expected insurance payment is
$16 - [2(.005) + 3(.01) + 4(.015) + d(.97)] = 15.9 - .97d$.
Setting this equal to 11.25 and solving for d results in $d = 4.7938$.