EXAM P QUESTIONS OF THE WEEK

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Week of April 30/07

Let X and Y be continuous random variables with joint density function

$f(x,y) = \begin{cases} c(y-x) \text{ for } 0<x<y<1 \\ 0, \text{otherwise} \end{cases}$.

What is the mean of the marginal distribution of X?

A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{3}{8}$ D) $\frac{1}{2}$ E) $\frac{5}{8}$

The solution can be found below.

Week of April 30/07 - Solution

In order for this to be a properly defined joint pdf, we must have
$\int_0^1 \int_x^1 c(y-x)\,dy\,dx = 1$.

$\int_x^1 c(y-x)\,dy = c[\frac{1-x^2}{2} - x(1-x)] = \frac{c(1-x)^2}{2}$,

and $\int_0^1 \frac{c(1-x)^2}{2}\,dx = \frac{c}{6}$.

Therefore, $c = 6$.

$f_X(x) = \int_x^1 6(y-x)\,dy = 3(1-x)^2$, $0 < x < 1$

$E[X] = \int_0^1 3x(1-x)^2\,dx = 3\int_0^1 [x - 2x^2 + x^3]\,dx = 3[\frac{1}{2} - 2(\frac{1}{3}) + \frac{1}{4}] = .25$.

Answer: B