EXAM P QUESTION OF THE WEEK

S. Broverman, 2008

Week of April 7/08

X has a uniform distribution on the interval $[0, 2]$
and
Y has a uniform distribution on the interval $[1, 3]$
and
X and Y are independent random variables.

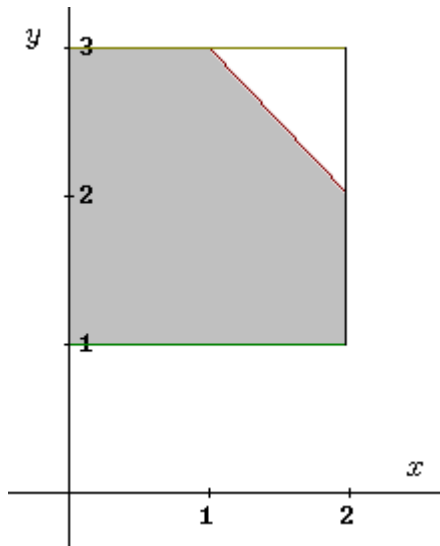$Z = X + Y$. Find the probability that Z is less than 4.

The solution can be found below.

Week of April 7/08 - Solution

If $X < 1$ then for any Y in $[0, 3]$ is true that $X + Y < 4$.
If $1 \leq X \leq 2$, then $X + Y < 4$ if $Y < 4 - X$.

The region of probability is the shaded region in the following graph.

$$P(X + Y < 4) = \int_0^1 \int_1^3 f(x, y)\, dy\, dx + \int_1^2 \int_1^{4-x} f(x, y)\, dy\, dx$$

The pdf of the joint distribution of X and Y is
$$f(x, y) = f_X(x) \cdot f_Y(y) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}.$$

$P(X + Y < 4)$ is $\frac{1}{4}$ of the shaded area. The shaded area is
$2 \times 2 - \frac{1}{2}(1 \times 1) = \frac{7}{2}$, so $P(X + Y < 4) = \frac{1}{4} \cdot \frac{7}{2} = \frac{7}{8}$.