EXAM P QUESTIONS OF THE WEEK

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Week of December 31/07

20% of the people living in a certain region with a large population were born outside the region, and 75% of those born outside the region have a first language other than English. Of those people living in the region who were born in the region, 90% have English as their first language.

100 people are chosen at random from the region. Use the normal approximation with continuity correction to find the approximate probability that in that group of 100 people the actual number of people who have English as their first language is equal to the expected number.

The solution can be found below.

Week of December 31/07 - Solution

For a randomly chosen person, the probability that the first language is English is

P(first language is English $\cap$ born outside the region)
$+ \ P$(first language is English $\cap$ born inside the region)
$= \ P$(first language is English|born outside the region) $\times P$(born outside the region)
$+ \ P$(first language is English|born inside the region) $\times P$(born inside the region)
$= (.25)(.2) + (.9)(.8) = .77$.

The number of people out 100 randomly chosen people from the region who have English as their first language, say N, has a binomial distribution with mean $100(.77) = 77$ and variance $100(.77)(.23) = 17.71$. Using the continuity correction, the normal approximation to the probability $P(N = 77)$ is
$$P(76.5 < N \le 77.5) = P(\frac{76.5-77}{\sqrt{17.71}} < \frac{N-77}{\sqrt{17.71}} \le \frac{77.5-77}{\sqrt{17.71}}) = \Phi(.12) - \Phi(-.12)$$
$$= .5478 - (1 - .5478) = .096$$