EXAM P QUESTIONS OF THE WEEK

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In the casino game of roulette, a wheel with 38 equally likely spots is spun, and a ball is dropped at random into one of the 38 spots. The 38 spots are numbers 1 to 36 along with 0 and 00.
On a spin of the wheel, a gambler can bet that the ball will drop into a specified spot. If the ball does drop into that spot, the gamble gets back the amount that he bet plus 36 time the amount that he bet. If that spot does not turn up, the gambler loses the amount bet. A gambler can also bet that the outcome of the spin will be even. If the ball drops into an even number spot from 2 to 36, the gambler gets back his bet plus an amount equal to the amount that he bet (the bet is lost if the spot is 0 or 00).

On every spin, Gambler 1 always bets that the ball will drop in the spot with the number 1, and Gambler 2 always bets that the ball will drop into an even numbered spot. X_1 denotes the net profit of Gambler 1 after n spins, and X_2 denotes the net profit of Gambler 2 after the n spins.

Find $E(X_2 - X_1)$.

A) $-\frac{n}{19}$ B) $-\frac{n}{38}$ C) 0 D) $\frac{n}{38}$ E) $\frac{n}{19}$

The solution can be found below.

Week of February 11/08 - Solution

Let Y denote the net profit of Gambler 1 for one spin.
Then Y is either -1 with probability $\frac{37}{38}$ or Y is 36 with probability $\frac{1}{38}$.
Then $E(Y) = -1 \times \frac{37}{38} + 36 \times \frac{1}{38} = -\frac{1}{38}$.
The net profit after n spins for Gambler 1 is $X_1 = Y_1 + Y_2 + \cdots + Y_n$,
and the expected profit is
$E(X_1) = E(Y_1) + E(Y_2) + \cdots + E(Y_n) = -\frac{1}{38} \times n = -\frac{n}{38}$.

Let Z denote the net profit of Gambler 2 for one spin.
Then Z is either -1 with probability $\frac{20}{38}$ or Z is 1 with probability $\frac{18}{38}$.
Then $E(Z) = -1 \times \frac{20}{38} + 1 \times \frac{18}{38} = -\frac{2}{38}$.
The net profit after n spins for Gambler 2 is $X_2 = Z_1 + Z_2 + \cdots + Z_n$,
and the expected profit is
$E(X_2) = E(Z_1) + E(Z_2) + \cdots + E(Z_n) = -\frac{2}{38} \times n = -\frac{2n}{38}$.

Then $E(X_2 - X_1) = E(X_2) - E(X_1) = -\frac{2n}{38} - (-\frac{n}{38}) = -\frac{n}{38}$

Answer: B