EXAM P QUESTIONS OF THE WEEK

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Week of February 12/07

X and Y have a bivariate normal distribution, and X and Y each have marginal distributions that are standard normal (mean 0, variance 1).

You are given $P(X > Y + 1) = .2119$.

Find $P(X > Y + 2)$.

The solution can be found below.

Week of February 12/07 - Solution

Suppose that the covariance between X and Y is C. Then $X - Y$ has a normal distribution with mean $1 - 1 = 0$ and variance
$Var[X - Y] = Var[X] + Var[Y] - 2\,Cov(X, Y) = 1 + 1 - C = 2 - 2C$.

Then, $P(X - Y > 1) = P(\frac{X-Y}{\sqrt{2-2C}} > \frac{1}{\sqrt{2-2C}}) = .2119$.

$Z = \frac{X-Y}{\sqrt{2-2C}}$ has a standard normal distribution, and from the standard normal table, we get $\frac{1}{\sqrt{2-2C}} = .80$.

Then, $P(X > Y + 2) = P(X - Y > +2) = P(\frac{X-Y}{\sqrt{2-2C}} > \frac{2}{\sqrt{2-2C}}) = P(Z > 1.6) = .0548.$