EXAM P QUESTIONS OF THE WEEK

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Week of February 18/08

The pdf of X is $f(x) = ax + b$ on the interval $[0, 2]$ and the pdf is 0 elsewhere.

You are given that the median of X is 1.25.

Find the variance of X.

The solution can be found below.

Week of February 18/08 - Solution

Since $f(x)$ is a pdf, we know that $\int_0^2 f(x)\,dx = 2a + 2b = 1$.

$F(x) = \int_0^t f(t)\,dt = \frac{at^2}{2} + bt$, so $F(\frac{5}{4}) = \frac{25a}{32} + \frac{5b}{4} = \frac{1}{2}$.

Solving these two equations results in $a = \frac{4}{15}$, $b = \frac{7}{30}$.

The mean of X is $E(X) = \int_0^2 x(\frac{4x}{15} + \frac{7}{30})\,dx = \frac{53}{45}$
and the second moment of X is $E(X^2) = \int_0^2 x^2(\frac{4x}{15} + \frac{7}{30})\,dx = \frac{76}{45}$.

The variance of X is $E(X^2) - [E(X)]^2 = \frac{76}{45} - \left(\frac{53}{45}\right)^2 = \frac{611}{45^2} = .302$.