EXAM P QUESTIONS OF THE WEEK

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Week of January 21/08

You are given the events A and B satisfy the relationship
$$P(A|B) = P(B|A) \quad \text{(conditional probabilities)}.$$

How many of the following statements always must be true?

I. A and B are independent.

II. $P(A) = P(B)$

III. $A \cap B = \emptyset$

A) None   B) Exactly 1   C) Exactly 2   D) Exactly 3
E) None of A,B,C or D is correct

The solution can be found below.

Week of January 21/08 - Solution

$P(A|B) = \frac{P(A \cap B)}{P(A)}$ and $P(B|A) = \frac{P(A \cap B)}{P(B)}$.

From $P(A|B) = P(B|A)$, we get $\frac{P(A \cap B)}{P(A)} = \frac{P(A \cap B)}{P(B)}$.

If $A \cap B \neq \emptyset$ then II is true, and if $A \cap B = \emptyset$ then III is true,
so it follows that A cannot be correct.

If $B = A \neq \emptyset$, then I is false, II is true and III is false, so exactly 1 statement is true.

If $P(A) = \frac{1}{2}$ and if $B = A'$, then I is false, II is true and II is true, so exactly 2 statements
are true.

If $A = B = \emptyset$, then I, II and III are true, so exactly 3 statements are true.

We have found combinations of events A and B for which answers B, C and D are not correct.
Therefore none of A, B, C or D is correct.   Answer: E