

EXAM P QUESTION OF THE WEEK

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Week of March 10/08

You are given the following information about events A , B and C :

- $P(A|C) = .5$
- $P(A|C') = .3$
- $P(C|B) = .625$
- A and B are conditionally independent of C and of C' , so that
 $P(A \cap B|C) = P(A|C) \times P(B|C)$ and $P(A \cap B|C') = P(A|C') \times P(B|C')$.

Find $P(A|B)$.

The solution can be found below.

Week of March 10/08 - Solution

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B \cap C) + P(A \cap B \cap C')}{P(B)}$$

$$\begin{aligned} P(A \cap B \cap C) &= P(A \cap B|C) \times P(C) = P(A|C) \times P(B|C) \times P(C) \\ &= P(A|C) \times P(B \cap C) \end{aligned}$$

and

$$\begin{aligned} P(A \cap B \cap C') &= P(A \cap B|C') \times P(C') = P(A|C') \times P(B|C') \times P(C') \\ &= P(A|C') \times P(B \cap C') \end{aligned}$$

$$\begin{aligned} \text{Then, } P(A|B) &= \frac{P(A|C) \times P(B \cap C) + P(A|C') \times P(B \cap C')}{P(B)} \\ &= P(A|C) \times P(C|B) + P(A|C') \times P(C'|B) \\ &= (.5)(.625) + (.3)(1 - .625) = .425 \end{aligned}$$