EXAM P QUESTIONS OF THE WEEK

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Week of March 19/07

Two players put one dollar into a pot. They decide to throw a pair of dice alternately. The first one who throws a total of 5 on both dice wins the pot. How much should the player who starts add to the pot to make this a fair game?

A) $\frac{9}{17}$ B) $\frac{8}{17}$ C) $\frac{1}{8}$ D) $\frac{2}{9}$ E) $\frac{8}{9}$

The solution can be found below.

Week of March 19/07 - Solution

Player 1 throws the dice on throws 1, 3, 5, ... and the probability that player wins on throw 2k + 1 is $(\frac{8}{9})^{2k} \cdot \frac{1}{9}$ for k = 0, 1, 2, 3, ... (there is a $\frac{1}{9}$ probability of throwing a total of 5 on any one throw of the pair of dice). The probability that player 1 wins the pot is $\frac{1}{9} + (\frac{8}{9})^2 \cdot \frac{1}{9} + (\frac{8}{9})^4 \cdot \frac{1}{9} + \cdots = \frac{1}{9} \cdot \frac{1}{1 - (\frac{8}{9})^2} = \frac{9}{17}$. Player 2 throws the dice on throws 2, 4, 6, ... The probability that player 2 wins the pot on throw 2k is $(\frac{8}{9})^{2k-1} \cdot \frac{1}{9}$ for k = 1, 2, 3, ... and the probability that player 2 wins is $\frac{8}{9} \cdot \frac{1}{9} + (\frac{8}{9})^3 \cdot \frac{1}{9} + (\frac{8}{9})^5 \cdot \frac{1}{9} + \cdots = \frac{8}{9} \cdot \frac{1}{9} \cdot \frac{1}{1 - (\frac{8}{9})^2} = \frac{8}{17} = 1 - \frac{9}{17}$.

If player 1 puts 1 + c dollars into the pot, then his expected gain is $1 \cdot \frac{9}{17} - (1 + c) \cdot \frac{8}{17}$. and player 2's expected gain is $(1 + c) \cdot \frac{8}{17} - 1 \cdot \frac{9}{17}$

In order for the two players to have the same expected gain, we must have

 $1 \cdot \frac{9}{17} - (1+c) \cdot \frac{8}{17} = 0$, so that $c = \frac{1}{8}$ Answer: C