EXAM P QUESTIONS OF THE WEEK

S. Broverman, 2007

Week of May 7/07

According to NBA playoff statistics, if a team has won 3 games and lost 1 game out of the first 4 games during a "best of 7" playoff series, that team has an 80% chance of winning the series. Statistics also show that if a team has won 3 games and lost 1 game out of the first 4 games and then loses the 5th game, that team has an 65% chance of winning the series. Find the probability that a team that has won 3 games and lost 1 game out of the first 4 games will win the next game.

A) $\frac{2}{7}$ B) $\frac{3}{7}$ C) $\frac{4}{7}$ D) $\frac{5}{7}$ E) $\frac{6}{7}$

The solution can be found below.

Week of May 7/07 - Solution

We define the following events and probabilities:

W = team wins the best-of-7 series,

G = team loses game 5,

T = team wins 3 of the first 4 games,

q = probability team wins 5th game given that it has won 3 of the first 4 games.

Our objective is to find $q = P[G'|T]$.

We are given $P[W|T] = .8$ and $P[W|T \cap G] = .65$.

$P[W \cap G|T] = \frac{P[W \cap G \cap T]}{P[T]} = \frac{P[W \cap G \cap T]}{P[G \cap T]} \cdot \frac{P[G \cap T]}{P[T]}$

$= P[W|G \cap T] \cdot P[G|T] = (.65)(1-q)$.

$.8 = P[W|T] = P[W \cap G|T] + P[W \cap G'|T] = (.65)(1-q) + P[W \cap G'|T]$.

$P[W \cap G'|T] = \frac{P[W \cap G' \cap T]}{P[T]} = \frac{P[W \cap G' \cap T]}{P[G' \cap T]} \cdot \frac{P[G' \cap T]}{P[T]}$

$= P[W|G' \cap T] \cdot P[G'|T] = q$

(this is true, since $P[W|G' \cap T] = 1$, because winning 3 out of the first 4 and then winning the 5th game results in winning the series) .

Therefore, $.8 = (.65)(1-q) + q \rightarrow q = \frac{.15}{.35} = \frac{3}{7}$.

Answer: B

An alternative solution has been sent to me by Brian Summers, of Maryville U in St Louis, MO. His solution is as follows.

$P[W'|T] = .2$ and $P[W'|T \cap G] = .35$ (these are the complement of the given probabilities $P[W|T] = .8$ and $P[W|T \cap G] = .65$).

First note that $P[W' \cap T] = P[W' \cap G \cap T]$, because in order to win 3 of the first 4 games and lose the series, it must be true that the team loses the 5th (and all subsequent games).

Therefore, $.2 = P[W'|T] = \frac{P[W' \cap T]}{P[T]} = \frac{P[W' \cap G \cap T]}{P[T]} = \frac{P[W' \cap G \cap T]}{P[G \cap T]} \cdot \frac{P[G \cap T]}{P[T]}$

$= P[W'|T \cap G] \cdot P[G|T] = (.35) \cdot P[G|T]$. It follows that $P[G|T] = \frac{.2}{.35} = \frac{4}{7}$,

and then $q = P[G'|T] = 1 - \frac{4}{7} = \frac{3}{7}$.