EXAM P QUESTIONS OF THE WEEK

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Week of October 15/07

X has a distribution which is partly continuous and partly discrete.
X has a discrete point of probability at $X = 1$ with probability p,
where $0 < p < 1$.
On the interval $(0, 1)$ X has a constant density of $\frac{1-p}{2}$,
and on the interval $(1, 2)$ X has a constant density of $\frac{1-p}{2}$.
Find the variance of X in terms of p

The solution can be found below.

Week of October 15/07 - Solution

Since X has a symmetric distribution about the point $X = 1$,
it follows that $E[X] = 1$. The second moment of X is
$E[X^2] = \int_0^1 x^2 \cdot \frac{1-p}{2}\, dx + 1^2 \cdot p + \int_1^2 x^2 \cdot \frac{1-p}{2}\, dx$
$= \frac{1}{3} \cdot \frac{1-p}{2} + p + \frac{7}{3} \cdot \frac{1-p}{2} = \frac{4}{3} - \frac{p}{3}$.

The variance of X is $Var[X] = E[X^2] - (E[X])^2 = \frac{4}{3} - \frac{p}{3} - 1 = \frac{1-p}{3}$.