EXAM P QUESTIONS OF THE WEEK

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Week of October 22/07

X has the following pdf: $f(x) = \begin{cases} x - \frac{x^2}{2} & \text{if } 0 < x \le 1 \\ \frac{x^2}{2} - x + 1 & \text{if } 1 < x < 2 \end{cases}$, and 0 otherwise.

The random variable Y is defined as follows: $Y = X^2$.

Find $F_Y(2)$.

A) .33 B) .48 C) .55 D) .67 E) .80

The solution can be found below.

Week of October 22/07 - Solution

The cdf of X is $F_X(t) = \int_0^t f(x)\,dx = \begin{cases} \frac{t^2}{2} - \frac{t^3}{6} & \text{if } 0 < t \le 1 \\ \frac{1}{3} + \frac{t^3-1}{6} - \frac{t^2-1}{2} + t - 1 & \text{if } 1 < x < 2 \end{cases}$

The cdf of Y is $F_Y(y) = P(Y \le y) = P(X^2 \le y) = P(X \le \sqrt{y}) = F_X(\sqrt{2})$.
Since $1 < \sqrt{2} < 2$, we get
$F_Y(2) = F_X(\sqrt{2}) = \frac{1}{3} + \frac{(\sqrt{2})^3-1}{6} - \frac{(\sqrt{2})^2-1}{2} + \sqrt{2} - 1 = .55$.