S. BROVERMAN STUDY GUIDE

FOR THE

SOCIETY OF ACTUARIES
EXAM MLC

2009 EDITION

EXCERPTS

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INTRODUCTORY NOTE

This study guide is designed to help in the preparation for Exam MLC of the Society of Actuaries (the life contingencies and probability exam).

The material for Exam MLC is divided into one large topic and two smaller topics. The large topic is life contingencies, and the smaller topics are Poisson processes and multi-state transition (Markov Chain) models. I think that the proper order in which to study the topics is the order in which they are listed in the previous sentence.

The study guide is divided into two volumes. Volume 1 consists of review notes, examples and problem sets for life contingencies. Volume 2 covers the other topics with review notes, examples and problem sets. Volume 2 also contains 12 practice exams of 30 questions each along with the May 2007 MLC exam and solutions. There are over 160 examples, over 300 problems in the problem sets and 360 questions in the 12 practice exams and May 2007 SOA exam. All of these (about 850) questions have detailed solutions. The notes are broken up into sections (32 sections for life contingencies, and one section each for Poisson Processes and Markov Chains). Each section has a suggested time frame.

Most of the examples in the notes and almost half of the problems in the problem sets are from older SOA or CAS exams (pre-2007) on the relevant topics. The 12 practice exams in Volume 2 include many questions from SOA exams released from 2000 to 2006. The practice exams have 30 questions each and are designed to be similar to actual 3-hour exams. The SOA and CAS questions are copyrighted by the SOA and CAS, and I gratefully acknowledge that I have been permitted to include them in this study guide.

Because of the time constraint on the exam, a crucial aspect of exam taking is the ability to work quickly. I believe that working through many problems and examples is a good way to build up the speed at which you work. It can also be worthwhile to work through problems that have been done before, as this helps to reinforce familiarity, understanding and confidence. Working many problems will also help in being able to more quickly identify topic and question types. I have attempted, wherever possible, to emphasize shortcuts and efficient and systematic ways of setting up solutions. There are also occasional comments on interpretation of the language used in some exam questions. While the focus of the study guide is on exam preparation, from time to time there will be comments on underlying theory in places that I feel those comments may provide useful insight into a topic.

It has been my intention to make this study guide self-contained and comprehensive for all Exam MLC topics, but there are occasional references to the books listed in the SOA exam catalog. While the ability to derive formulas used on the exam is usually not the focus of an exam question, it is useful in enhancing the understanding of the material and may be helpful in memorizing formulas. There may be an occasional reference in the review notes to a derivation, but you are encouraged to review the official reference material for more detail on formula derivations.

In order for the review notes in this study guide to be most effective, you should have some background at the junior or senior college level in probability and statistics. It will be assumed that you are reasonably familiar with differential and integral calculus.
Of the various calculators that are allowed for use on the exam, I think that the BA II PLUS is probably the best choice. It has several memories and has good financial functions. I think that the TI-30X IIS would be the second best choice.

There is a set of tables that has been provided with the exam in past sittings. These tables consist of a standard normal distribution probability table and a life table. The tables should be available for download from the Society of Actuaries website. It is recommended that you have them available while studying.

Based on the weight applied to topics on recent actual exams, I have created the practice exams to include about 24 questions on life contingencies and 3 each on Poisson processes and multi-state transition models.

If you have any questions, comments, criticisms or compliments regarding this study guide, you may contact me at the address below. I apologize in advance for any errors, typographical or otherwise, that you might find, and it would be greatly appreciated if you would bring them to my attention. I will be maintaining a website for errata that can be accessed from www.sambroverman.com . It is my sincere hope that you find this study guide helpful and useful in your preparation for the exam. I wish you the best of luck on the exam.

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LIFE CONTINGENCIES SECTION 26
THE LAST SURVIVOR STATUS AND THE COMMON SHOCK MODEL

The suggested time frame for covering this section is 2 hours.

The last-survivor status (Bowers 9.4)

$T(\overline{xy}) = max[T(x), T(y)]$ is the time until the second death of the pair of lives aged (x) and (y); the last-survivor status fails at the time of the second death ($T(\overline{xy})$ is the largest order statistic of the pair of random variables $T(x)$ and $T(y)$). One basic probability for the last survivor status is ${}_tq_{\overline{xy}}$, the probability that the status fails by time t. The last survivor status fails by time t if the second death has occurred by time t, which is the same as saying that both deaths have occurred by time t. Therefore, ${}_tq_{\overline{xy}} = P[(T(x) \leq t) \cap (T(y) \leq t)]$. Note that ${}_tp_{\overline{xy}} = 1 - {}_tq_{\overline{xy}}$ is the probability that not both have died by time t, or in other words it is the probability that at least one (or both) have survived to time t.

There is a general "theme" that arises for the formulations of functions involving the last survivor status: $g(\overline{xy}) = g(x) + g(y) - g(xy)$, where the function g can be a probability, expectation, density, annuity or insurance. This theme is illustrated in the following formulations. Many of these relationships are variations on the rule
$P[A \cup B] = P[A] + P[B] - P[A \cap B]$, which can be written
$P[A \cup B] + P[A \cap B] = P[A] + P[B]$.

If A is the event that (x) dies by time t, and B is the event that (y) dies by time t, then
$P[A] = P[T(x) \leq t] = {}_tq_x$ and $P[B] = P[T(y) \leq t] = {}_tq_y$.
We then have $P[A \cup B] = P[(T(x) \leq t) \cup (T(y) \leq t)] = P[T(xy) \leq t] = {}_tq_{xy}$
(this is the probability that at least one of (x) and (y) dies by time t).
We also have $P[A \cap B] = P[(T(x) \leq t) \cap (T(y) \leq t)] = P[T(\overline{xy}) \leq t] = {}_tq_{\overline{xy}}$
(this is the probability that both die by time t).
We then get $P[A \cup B] + P[A \cap B] = {}_tq_{xy} + {}_tq_{\overline{xy}} = {}_tq_x + {}_tq_y = P[A] + P[B]$.
This is more likely to be written in the form ${}_tq_{\overline{xy}} = {}_tq_x + {}_tq_y - {}_tq_{xy}$.

This reasoning applies to many functions for the last survivor status. We also have

$$T(xy) + T(\overline{xy}) = T(x) + T(y) \quad , \quad T(xy) \cdot T(\overline{xy}) = T(x) \cdot T(y)$$

Last survivor status relationships

$\boldsymbol{{}_tq_{xy} + {}_tq_{\overline{xy}} = {}_tq_x + {}_tq_y} = F_{T(xy)}(t) + F_{T(\overline{xy})}(t) = F_{T(x)}(t) + F_{T(y)}(t)$,

from which we get $\boldsymbol{{}_tq_{\overline{xy}} = {}_tq_x + {}_tq_y - {}_tq_{xy}}$

$\boldsymbol{{}_tp_{xy} + {}_tp_{\overline{xy}} = {}_tp_x + {}_tp_y}$, from which we get $\boldsymbol{{}_tp_{\overline{xy}} = {}_tp_x + {}_tp_y - {}_tp_{xy}}$

${}_tq_{\overline{xy}} = F_{T(\overline{xy})}(t) = F_{T(x)}(t) + F_{T(y)}(t) - F_{T(xy)}(t) = F_{T(x)T(y)}(t,t)$
$= P[T(x) \leq t \cap T(y) \leq t]$ (this is the probability that both die before time t)

$\boldsymbol{{}_{t|u}q_{\overline{xy}} = {}_{t+u}q_{\overline{xy}} - {}_tq_{\overline{xy}} = {}_tp_{\overline{xy}} - {}_{t+u}p_{\overline{xy}} = {}_{t|u}q_x + {}_{t|u}q_y - {}_{t|u}q_{xy}}$
(this is the probability that the second death is after time t and before time $t+u$)

${}_tp_{\overline{xy}}\mu_{\overline{xy}}(t) = f_{T(\overline{xy})}(t) = f_{T(x)}(t) + f_{T(y)}(t) - f_{T(xy)}(t)$
$= {}_tp_x\mu(x+t) + {}_tp_y\mu(y+t) - {}_tp_{xy}\mu_{xy}(t)$

$\mu_{\overline{xy}}(t) = \frac{{}_tp_x\mu(x+t) + {}_tp_y\mu(y+t) - {}_tp_{xy}\mu_{xy}(t)}{{}_tp_x + {}_tp_y - {}_tp_{xy}}$
(note that $\mu_{\overline{xy}}(t)$ doesn't satisfy the $g(\overline{xy}) = g(x) + g(y) - g(xy)$ relationship)

$c^{T(xy)} + c^{T(\overline{xy})} = c^{T(x)} + c^{T(y)}$ for any constant c

$\boldsymbol{\mathring{e}_{\overline{xy}} = \int_0^\infty {}_tp_{\overline{xy}}\,dt = \int_0^\infty ({}_tp_x + {}_tp_y - {}_tp_{xy})\,dt = \mathring{e}_x + \mathring{e}_y - \mathring{e}_{xy}}$.
(∞ in this integral is the larger of the times until ω for (x) and (y))

$K(\overline{xy})$ is the completed number of years until the second death, so that
$P[K(\overline{xy}) = k] = f_{K(\overline{xy})}(k) = P[k < T(\overline{xy}) \leq k+1]$
$= \boldsymbol{{}_{k|}q_{\overline{xy}} = {}_{k+1}q_{\overline{xy}} - {}_kq_{\overline{xy}} = {}_kp_{\overline{xy}} - {}_{k+1}p_{\overline{xy}} = {}_{k|}q_x + {}_{k|}q_y - {}_{k|}q_{xy}}$
$= {}_kp_xq_{x+k} + {}_kp_yq_{y+k} - {}_kp_{xy}q_{x+k:y+k}$

$$\boldsymbol{e_{\overline{xy}} = \sum_{k=1}^{\infty} {}_kp_{\overline{xy}} = \sum_{k=1}^{\infty} ({}_kp_x + {}_kp_y - {}_kp_{xy}) = e_x + e_y - e_{xy}}$$

Note that ${}_tp_{\overline{xy}} \neq {}_np_{\overline{xy}} \cdot {}_{t-n}p_{\overline{x+n:y+n}}$. The factorization does not work in the last-survivor case.

Note also that $T(xy)$ and $T(\overline{xy})$ are never independent, since $T(xy) \leq T(\overline{xy})$ always.
The derivation of the following identity is given in the textbook:

$Cov[T(xy), T(\overline{xy})] = Cov[T(x), T(y)] + (\overset{\circ}{e}_x - \overset{\circ}{e}_{xy})(\overset{\circ}{e}_y - \overset{\circ}{e}_{xy})$

IMPORTANT NOTE: **if $T(x)$ and $T(y)$ are independent**, then $\boldsymbol{{}_tq_{\overline{xy}} = {}_tq_x \cdot {}_tq_y}$.
This is true since

${}_tq_{\overline{xy}} = P[(T(x) \leq t) \cap (T(y) \leq t)] = P[T(x) \leq t] \cdot P[T(y) \leq t] = {}_tq_x \cdot {}_tq_y$.

This relationship is used to simplify other probability relationships when independence is assumed. Also, under the assumption of independence we have

${}_{k|}q_{\overline{xy}} = {}_kq_x \; {}_kp_y \; q_{y+k} + {}_kq_y \; {}_kp_x \; q_{x+k} + {}_kp_x \; {}_kp_y \; q_{x+k} \; q_{y+k}$, and

$Cov[T(xy), T(\overline{xy})] = (\overset{\circ}{e}_x - \overset{\circ}{e}_{xy})(\overset{\circ}{e}_y - \overset{\circ}{e}_{xy})$ (since $Cov[T(x), T(y)] = 0$ because of independence) .

When finding complete or curtate expectations for the last survivor status, it is usually more convenient to use the form $\overset{\circ}{e}_x + \overset{\circ}{e}_y - \overset{\circ}{e}_{xy}$ instead of $\overset{\circ}{e}_{\overline{xy}}$. The following example illustrates this.

Example LC-118: Smith and Jones are independent lives aged 90 and 95, respectively, and both have mortality that follows DeMoivre's Law with $\omega = 100$. Find $\overset{\circ}{e}_{\overline{90:95}}$.
Solution: $\overset{\circ}{e}_{\overline{90:95}} = \int_0^\infty {}_tp_{\overline{90:95}} \, dt$ and ${}_tp_{\overline{90:95}}$ is the probability that at least one has survived to time t. Since (90) has 10 years until the end of his survival distribution, but (95) only has 5 years, ${}_tp_{\overline{90:95}}$ becomes 0 for $t \geq 10$, but is non-zero for $t < 10$. Therefore, an appropriate upper limit for the integral is 10 in this case, $\overset{\circ}{e}_{\overline{90:95}} = \int_0^{10} {}_tp_{\overline{90:95}} \, dt$. There are a couple of ways we can proceed:

(i) ${}_tp_{\overline{90:95}} = 1 - {}_tq_{\overline{90:95}} = 1 - ({}_tq_{90})({}_tq_{95})$ (the last equality follows from independence of (90) and (95) . Then $\overset{\circ}{e}_{\overline{90:95}} = \int_0^{10} [1 - ({}_tq_{90})({}_tq_{95})] \, dt$. This becomes a little awkward, since ${}_tq_{95} = 1$ for $t \geq 5$; we would split the integral into two parts.

$$\overset{\circ}{e}_{\overline{90:95}} = \int_0^5 [1 - ({}_tq_{90})({}_tq_{95})] \, dt + \int_5^{10} [1 - ({}_tq_{90})({}_tq_{95})] \, dt$$
$$= \int_0^5 [1 - (\tfrac{t}{10})(\tfrac{t}{5})] \, dt + \int_5^{10} [1 - (\tfrac{t}{10})(1)] \, dt = 4.167 + 1.25 = 5.417 \; .$$

Example LC-118 continued

(ii) $_tp_{\overline{90:95}} = {}_tp_{90} + {}_tp_{95} - {}_tp_{90:95} = {}_tp_{90} + {}_tp_{95} - ({}_tp_{90})({}_tp_{95})$.

Then $\mathring{e}_{\overline{90:95}} = \int_0^\infty [{}_tp_{90} + {}_tp_{95} - ({}_tp_{90})({}_tp_{95})]\,dt = \mathring{e}_{90} + \mathring{e}_{95} - \mathring{e}_{90:95}$.

Under DeMoivre's Law with $\omega = 100$, $\mathring{e}_{90} = \frac{100-90}{2} = 5$ and $\mathring{e}_{95} = \frac{100-95}{2} = 2.5$.

Also, $\mathring{e}_{90:95} = \int_0^\infty {}_tp_{90:95}\,dt = \int_0^5 ({}_tp_{90})({}_tp_{95})\,dt$ (for joint-life status expectation, upper limit of integral is earliest time until death for the two individuals) . Then

$\mathring{e}_{90:95} = \int_0^5 (\frac{10-t}{10})(\frac{5-t}{5})\,dt = 2.083$, and $\mathring{e}_{\overline{90:95}} = 5 + 2.5 - 2.083 = 5.417$.

In general, approach (ii) is a little more straightforward and avoids separating the integral into two parts. The last survivor expectation can always be written in the form $\mathring{e}_{\overline{xy}} = \mathring{e}_x + \mathring{e}_y - \mathring{e}_{xy}$, where $\mathring{e}_x$ and $\mathring{e}_y$ are found in the usual way, and $\mathring{e}_{xy}$ is found as above or as in Example LC-116 earlier in these notes. □

Example LC-119 (SOA): You are given:

(i) $_5p_{50} = 0.9$ (ii) $_5p_{60} = 0.8$ (iii) $q_{55} = 0.03$ (iv) $q_{65} = 0.05$

(v) $T(50)$ and $T(60)$ are independent.

Calculate $_{5|}q_{\overline{50:60}}$.

Solution: Under the assumption of independence, the main simplifying relationships for two life statuses are $_tp_{xy} = {}_tp_x \cdot {}_tp_y$ for the joint life status, and $_tq_{\overline{xy}} = {}_tq_x \cdot {}_tq_y$ for the last-survivor status. In this case, it is probably most efficient to write

$_{5|}q_{\overline{50:60}} = {}_6q_{\overline{50:60}} - {}_5q_{\overline{50:60}} = {}_6q_{50} \cdot {}_6q_{60} - {}_5q_{50} \cdot {}_5q_{60}$.

From the given information, $_6p_{50} = {}_5p_{50} \cdot p_{55} = (.9)(.97) = .873$, and $_6p_{60} = (.8)(.95) = .76$, so that $_{5|}q_{\overline{50:60}} = (.127)(.24) - (.1)(.2) = .01048$. □

Example LC-120 (SOA): You are given:
(i) $T(x)$ and $T(y)$ are independent.
(ii) $E[T(x)] = E[T(y)] = 4.0$ (iii) $Cov[T(xy), T(\overline{xy})] = 0.09$
Calculate $E[T(xy)]$.

Solution: The general form for $Cov[T(xy), T(\overline{xy})]$ is

$Cov[T(xy), T(\overline{xy})] = Cov[T(x), T(y)] + (\mathring{e}_x - \mathring{e}_{xy})(\mathring{e}_y - \mathring{e}_{xy})$, and if $T(x)$ and $T(y)$ are independent, then $Cov[T(x), T(y)] = 0$ so that $Cov[T(xy), T(\overline{xy})] = (\mathring{e}_x - \mathring{e}_{xy})(\mathring{e}_y - \mathring{e}_{xy})$.

Therefore, $.09 = (4 - \mathring{e}_{xy})(4 - \mathring{e}_{xy}) = (4 - \mathring{e}_{xy})^2 \rightarrow 4 - \mathring{e}_{xy} = \pm .3 \rightarrow \mathring{e}_{xy} = 3.7$ or 4.3 .

Since $T(xy) = min\{T(x), T(y)\}$, it follows that $\mathring{e}_{xy} \leq \mathring{e}_x$, and therefore, we choose the smaller root, $\mathring{e}_{xy} = 3.7$ (note that $\mathring{e}_{\overline{xy}} = 4.3$). □

The Common Shock Dependent Lifetime Model (Bowers 9.6.1)

In the formulation of the joint distribution of the continuous random variables $T(x)$ and $T(y)$, the probability that $T(x)$ and $T(y)$ die at the same instant is 0. The reason for this is similar to the reason that for any continuous random variable W we have $P[W = a] = 0$. For a continuous random variable W, we can only have probabilities over an interval;
$P[a < W < b] = \int_a^b f_W(w)\, dw$. If we try to find $P[W = a]$ we get $\int_a^a f_W(w)\, dw = 0$.
In a similar way, for a pair of random variables W and U, we can only get non-zero probabilities on a two dimensional region, $P[(a < W < b) \cap (c < U < d)] = \int_a^b \int_c^d f_{W,U}(w,u)\, du\, dw$.
For a joint pair of continuous random variables W and U, if we try to find a probability on a one-dimension region such as $P[W = U]$ (represented by a straight line in two-dimensional space), we get $P[W = U] = 0$.

To allow for the possibility of simultaneous deaths of two (or more) lives, a **common shock random variable** is introduced (representing some catastrophe such as auto accident that could claim both lives simultaneously). $T^*(x)$ and $T^*(y)$ denote independent lifetimes for (x) and (y) in the absence of a common shock. An alternative explanation is this; if there was no common shock, $T^*(x)$ would just be $T(x)$, and the same for (y). The common shock is the hazard that (x) and (y) share, and $T^*(x)$ would be the time until (x)'s death if the common hazard was eliminated (same for y).

Z denotes the time until the occurrence of a common shock, also assumed to be independent of $T^*(x)$ and $T^*(y)$. Z is assumed to have an exponential distribution with $s_Z(z) = e^{-\lambda z}$, where $z > 0$ and $\lambda \geq 0$. If we consider (x)'s survival alone, then (x) will die either due to the common shock, or to a cause other than common shock. Therefore, the time until (x)'s death is the earlier of the occurrence of either the common shock or a death event other than common shock, so that $T(x) = min[T^*(x), Z]$. The same is true for (y), $T(y) = min[T^*(y), Z]$.

In order for (x) to survive s years it must be true that the common shock has not occurred and (x) has not died as a result any other cause either. Therefore, the survival function for the marginal distribution of $T(x)$ is ${}_sp_x = P[T(x) > s] = P[(T^*(x) > s) \cap (Z > s)]$.
Since $T^*(x)$ and Z are independent, we have
$P[(T^*(x) > s) \cap (Z > s)] = P[T^*(x) > s] \cdot P[Z > s] = s_{T^*(x)}(s) \cdot e^{-\lambda s} = {}_sp_x^* \cdot e^{-\lambda s}$.
We can write ${}_sp_x$ as $\boldsymbol{{}_sp_x = P[T(x) > s] = s_{T^*(x)}(s) \cdot e^{-\lambda s} = {}_sp_x^* \cdot e^{-\lambda s}}$.
In a similar way, $\boldsymbol{{}_tp_y = s_{T(y)}(t) = s_{T^*(y)}(t) \cdot e^{-\lambda t}} = \boldsymbol{{}_sp_y^* \cdot e^{-\lambda s} = P[T(y) > t]}$.
In this notation, ${}_sp_x^* = s_{T^*(x)}(s)$ denotes the probability that, ignoring common shock, x does not die by time s due to any other causes.

For (x) and (y), the joint-life status survives to time t if both survive to time t. This means that the common shock has not occurred by time t, and neither of (x) nor (y) has died due to any other cause. Therefore, we have ${}_tp_{xy} = P[(T^*(x) > s) \cap (T^*(y) > t) \cap (Z > s)]$.
Since $T^*(x)$, $T^*(y)$ and Z are mutually independent, if follows that

$$\boldsymbol{{}_tp_{xy}} = P[T^*(x) > s] \cdot P[T^*(y) > t] \cdot P[Z > s]$$

$$= \boldsymbol{s_{T^*(x)}(t) \cdot s_{T^*(y)}(t) \cdot e^{-\lambda t} = {}_tp_x^* \cdot {}_tp_y^* \cdot e^{-\lambda t}} .$$

Note that ${}_tp_{xy} = P[(T(x) > s) \cap (T(y) > t)]$ is always true, but in this case $T(x)$ and $T(y)$ are not independent, since they both are related to the common shock random variable Z. **The exam questions that have contained a reference to the common shock model will likely use constant force assumptions for $T^*(x)$ and $T^*(y)$, say μ_x^* and μ_y^* . In that case, we have $\boldsymbol{{}_tp_x^* = e^{-t\mu_x^*}}$, $\boldsymbol{{}_tp_y^* = e^{-t\mu_y^*}}$, and $\boldsymbol{{}_tp_x = e^{-t(\mu_x^*+\lambda)}}$, $\boldsymbol{{}_tp_y = e^{-t(\mu_y^*+\lambda)}}$,
and $\boldsymbol{{}_tp_{xy} = e^{-t(\mu_x^*+\mu_y^*+\lambda)}}$.**

The density function for simultaneous death of (x) and (y) at time t is

$$f_{T(x)T(y)}(t,t) = \lambda e^{-\lambda t} \cdot s_{T^*(x)}(t) \cdot s_{T^*(y)}(t) .$$

The probability that (x) and (y) die as a result of common shock within n years is

$\int_0^n \lambda e^{-\lambda t} \cdot s_{T^*(x)}(t) \cdot s_{T^*(y)}(t)\, dt$.

With $n = \infty$, in the constant force case, this is equal to $\frac{\lambda}{\mu_x^* + \mu_y^* + \lambda}$.

The probability that (x) survives to time s and (y) survives to time t (and neither has died due to the common shock, or for any other reason) is $s_{T(x)T(y)}(s,t) = s_{T^*(x)}(s)\ s_{T^*(y)}(t)\ e^{-\lambda[max(s,t)]}$.

The density function of the joint distribution of $T(x)$ and $T(y)$ is

$f_{T(x)T(y)}(s,t) = \frac{\partial^2}{\partial s\, \partial t} s_{T(x)T(y)}(s,t) = \frac{\partial^2}{\partial s\, \partial t} s_{T^*(x)}(s)\ s_{T^*(y)}(t)\ e^{-\lambda[max(s,t)]}$.

The last-survivor status survival function is

${}_tp_{\overline{xy}} = {}_tp_x + {}_tp_y - {}_tp_{xy} = [s_{T^*(x)}(t) + s_{T^*(y)}(t) - s_{T^*(x)}(t) \cdot s_{T^*(y)}(t)]e^{-\lambda t} = s_{T(\overline{xy})}(t)$.

Note that if $\lambda = 0$, then this common shock model reduces to the usual joint distribution of independent $T(x)$ and $T(y)$.

Example LC-121: (40) and (60) are lives subject to a common shock with $\lambda = .02$. In the absence of the common shock, (40) and (60) have independent lifetimes both following DeMoivre's Law with $\omega = 100$. Find (a) ${}_{10}p_{40}$ (b) ${}_{10}p_{40:60}$ (c) $\mathring{e}_{40}$ (d) $\mathring{e}_{40:60}$

Solution: (a) ${}_tp_{40} = s_{T^*(40)}(t) \cdot e^{-\lambda t} = (\frac{100-40-t}{100-40}) \cdot e^{-.02t} = (\frac{60-t}{60}) \cdot e^{-.02t}$

$\rightarrow {}_{10}p_{40} = (\frac{5}{6})e^{-.2} = .682$.

(b) ${}_{10}p_{40:60} = s_{T^*(40)}(10) \cdot s_{T^*(60)}(10) \cdot e^{-10\lambda} = (\frac{50}{60})(\frac{30}{40}) \cdot e^{-.2} = .512$.

(c) $\mathring{e}_{40} = \int_0^{60} {}_tp_{40}\, dt = \int_0^{60} (\frac{60-t}{60}) \cdot e^{-.02t}\, dt = \int_0^{60} [e^{-.02t} - \frac{t}{60} \cdot e^{-.02t}]\, dt$

The antiderivative of te^{-at} is $-\frac{t}{a}e^{-at} - \frac{1}{a^2}e^{-at}$, so the integral becomes

$\frac{-e^{-.02t}}{.02} + \frac{1}{60} \cdot \frac{t}{.02} \cdot e^{-.02t} + \frac{1}{60} \cdot \frac{1}{.0004} \cdot e^{-.02t} \Big|_{t=0}^{t=60} = 20.9$.

(d) $\mathring{e}_{40:60} = \int_0^{60} {}_tp_{40:60}\, dt = \int_0^{40} (\frac{60-t}{60}) \cdot (\frac{40-t}{40}) \cdot e^{-.02t}\, dt$

$= \frac{1}{2400} \cdot \int_0^{40} (2400 - 100t + t^2) \cdot e^{-.02t}\, dt$

The antiderivative of t^2e^{-at} is $-\frac{t^2}{a}e^{-at} - \frac{2t}{a^2}e^{-at} - \frac{2}{a^3}e^{-at}$, so the integral is

$= \frac{1}{2400} \cdot [\frac{-2400e^{-.02t}}{.02} + \frac{100te^{-.02t}}{.02} + \frac{100e^{-.02t}}{.0004} - \frac{t^2e^{-.02t}}{.02} - \frac{2te^{-.02t}}{.0004} - \frac{e^{-.02t}}{.000008} \Big|_{t=0}^{t=60}]$

$= 12.35$. □

LIFE CONTINGENCIES SECTION 26 - EXERCISES

1. (50) and (60) are independent lives with mortality based on the Illustrative Life Table. Find the probability that the second death occurs between 10 and 20 years from now.

2. Smith and Jones are independent lives aged 90 and 95, respectively, and both have mortality that follows DeMoivre's Law with $\omega = 100$. Find $\mathring{e}_{\overline{90:95}:\overline{7}|}$.

3. A common shock model has $\lambda = .02$, and $T^*(x)$ and $T^*(y)$ are both subject to a constant force of mortality of .01 . Find

(a) the probability that (x) and (y) will die within 10 years, and

(b) the probability that (x) and (y) will die within 10 years as a result of the common shock.

LIFE CONTINGENCIES SECTION 26 - SOLUTIONS TO EXERCISES

1. ${}_{10|10}q_{\overline{50:60}} = {}_{20}q_{\overline{50:60}} - {}_{10}q_{\overline{50:60}} = ({}_{20}q_{50})({}_{20}q_{60}) - ({}_{10}q_{50})({}_{10}q_{60})$

$= (1 - {}_{20}p_{50})(1 - {}_{20}p_{60}) - (1 - {}_{10}p_{50})(1 - {}_{10}p_{60}) = .120$.

2. $\mathring{e}_{\overline{90:95}:\overline{7}|} = \mathring{e}_{90:\overline{7}|} + \mathring{e}_{95:\overline{7}|} - \mathring{e}_{90:95:\overline{7}|}$.

$\mathring{e}_{90:\overline{7}|} = \int_0^7 {}_tp_{90}\,dt = \int_0^7 (1 - \frac{t}{10})\,dt = 4.55$,

$\mathring{e}_{95:\overline{7}|} = \int_0^7 {}_tp_{95}\,dt = \int_0^5 {}_tp_{95}\,dt = \int_0^5 (1 - \frac{t}{5})\,dt = 2.5$

$\mathring{e}_{90:95:\overline{7}|} = \int_0^7 {}_tp_{90:95}\,dt = \int_0^5 {}_tp_{90:95}\,dt = \int_0^5 (1 - \frac{t}{10})(1 - \frac{t}{5})\,dt = 2.083$,

$\mathring{e}_{\overline{90:95}:\overline{7}|} = 4.55 + 2.5 - 2.083 = 4.97$.

3.(a) ${}_{10}q_{\overline{xy}} = 1 - {}_{10}p_{\overline{xy}} = 1 - {}_{10}p_x - {}_{10}p_y + {}_{10}p_{xy} =$

$1 - (e^{-10(.01)})(e^{-10(.02)}) - (e^{-10(.01)})(e^{-10(.02)}) + (e^{-10(.01)})(e^{-10(.01)})(e^{-10(.02)}) = .189$.

(b) $\int_0^n \lambda e^{-\lambda t} \cdot s_{T^*(x)}(t) \cdot s_{T^*(y)}(t)\,dt = \int_0^{10} (.02) \cdot e^{-.02t} \cdot e^{-.01t} \cdot e^{-.01t}\,dt = .165$.

S. BROVERMAN MLC STUDY GUIDE
PRACTICE EXAM 1

1. For a fully discrete 3-year endowment insurance of 1000 on (x), you are given:

(i) ${}_kL$ is the prospective loss random variable at time k.

(ii) $i = 0.10$ (iii) $\ddot{a}_{x:\overline{3}|} = 2.70182$

(iv) Premiums are determined by the equivalence principle.

Calculate ${}_1L$, given that (x) dies in the second year from issue.

A) 540 B) 630 C) 655 D) 720 E) 910

2. For a double-decrement model:

(i) ${}_tp'^{(1)}_{40} = 1 - \frac{t}{60}$, $0 \le t \le 60$

(ii) ${}_tp'^{(2)}_{40} = 1 - \frac{t}{40}$, $0 \le t \le 40$

Calculate $\mu^{(\tau)}_{40}(20)$.

A) 0.025 B) 0.038 C) 0.050 D) 0.063 E) 0.075

3. For independent lives (35) and (45):

(i) ${}_5p_{35} = 0.90$ (ii) ${}_5p_{45} = 0.80$ (iii) $q_{40} = 0.03$ (iv) $q_{50} = 0.05$

Calculate the probability that the last death of (35) and (45) occurs in the 6th year.

A) 0.0095 B) 0.0105 C) 0.0115 D) 0.0125 E) 0.0135

4. For a fully discrete whole life insurance of 100,000 on (35) you are given:

(i) Percent of premium expenses are 10% per year.

(ii) Per policy expenses are 25 per year.

(iii) Per thousand expenses are 2.50 per year.

(iv) All expenses are paid at the beginning of the year.

(v) $1000P_{35} = 8.36$.

Calculate the level annual expense-loaded premium using the equivalence principle.

A) 930 B) 1041 C) 1142 D) 1234 E) 1352

5. Kings of Fredonia drink glasses of wine at a Poisson rate of 2 glasses per day. Assassins attempt to poison the king's wine glasses. There is a 0.01 probability that any given glass is poisoned. Drinking poisoned wine is always fatal instantly and is the only cause of death. The occurrences of poison in the glasses and number of glasses drunk are independent events. Calculate the probability that the current king survives at least 30 days.

A) 0.40 B) 0.45 C) 0.50 D) 0.55 E) 0.60

6. Z is the present-value random variable for a whole life insurance of b payable at the moment of death of (x). You are given:

(i) $\delta = 0.04$ (ii) $\mu_x(t) = 0.02\,,\ t \geq 0$

(iii) The single benefit premium for this insurance is equal to $Var(Z)$.

Calculate b.

A) 2.75 B) 3.00 C) 3.25 D) 3.50 E) 3.75

7. For a special 3-year term insurance on (30), you are given:

(i) Premiums are payable semiannually.

(ii) Premiums are payable only in the first year.

(iii) Benefits, payable at the end of the year of death, are:

k	b_{k+1}
0	1000
1	500
2	250

(iv) Mortality follows the Illustrative Life Table.

(v) Deaths are uniformly distributed within each year of age.

(vi) $i = 0.06$

Calculate the amount of each semiannual benefit premium for this insurance.

A) 1.3 B) 1.4 C) 1.5 D) 1.6 E) 1.7

8. For a Markov model with three states, Healthy (0), Disabled (1), and Dead (2):

(i) The annual transition matrix is given by

	0	1	2
0	0.70	0.20	0.10
1	0.10	0.65	0.25
2	0	0	1

(ii) There are 100 lives at the start, all Healthy. Their future states are independent.

Calculate the variance of the number of the original 100 lives who die within the first two years.

A) 11 B) 14 C) 17 D) 20 E) 23

9. An insurance company issues a special 3-year insurance to a high-risk individual. You are given the following homogeneous Markov chain model:

(i) State 1: active
State 2: disabled
State 3: withdrawn
State 4: dead

Transition probability matrix:

	1	2	3	4
1	0.4	0.2	0.3	0.1
2	0.2	0.5	0	0.3
3	0	0	1	0
4	0	0	0	1

(ii) Changes in state occur at the end of the year.

(iii) The death benefit is 1000, payable at the end of the year of death.

(iv) $i = 0.05$

(v) The insured is disabled at the end of year 1.

Calculate the actuarial present value of the prospective death benefits at the beginning of year 2.

A) 440 B) 528 C) 634 D) 712 E) 803

10. For a fully discrete whole life insurance of b on (x), you are given:

(i) $q_{x+9} = 0.02904$ (ii) $i = 0.03$

(iii) The initial benefit reserve for policy year 10 is 343.

(iv) The net amount at risk for policy year 10 is 872.

(v) $\ddot{a}_x = 14.65976$

Calculate the terminal benefit reserve for policy year 9.

A) 280 B) 288 C) 296 D) 304 E) 312

11. For a special fully discrete 2-year endowment insurance of 1000 on (x), you are given:

(i) The first year benefit premium is 668.

(ii) The second year benefit premium is 258.

(iii) $d = 0.06$

Calculate the level annual premium using the equivalence principle.

A) 469 B) 479 C) 489 D) 499 E) 509

12. For an increasing 10-year term insurance, you are given:

(i) $b_{k+1} = 100,000(1+k)$, $k = 0, 1, ..., 9$

(ii) Benefits are payable at the end of the year of death.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

(v) The single benefit premium for this insurance on (41) is 16,736.

Calculate the single benefit premium for this insurance on (40).

A) 12,700 B) 13,600 C) 14,500 D) 15,500 E) 16,300

13. For a fully discrete whole life insurance of 1000 on (x):

(i) Death is the only decrement.

(ii) The annual benefit premium is 80.

(iii) The annual contract premium is 100.

(iv) Expenses in year 1, payable at the start of the year, are 40% of contract premiums.

(v) $i = 0.10$ (vi) $1000\,{}_1V_x = 40$

Calculate the asset share at the end of the first year.

A) 17 B) 18 C) 19 D) 20 E) 21

14. A fully discrete 3-year term insurance of 10,000 on (40) is based on a double-decrement model, death and withdrawal:

(i) Decrement 1 is death. (ii) $\mu_{40}^{(1)}(t) = 0.02$, $t \geq 0$

(iii) Decrement 2 is withdrawal, which occurs at the end of the year.

(iv) $q_{40+k}^{\prime(2)} = 0.04$, $k = 0, 1, 2$ (v) $v = 0.95$

Calculate the actuarial present value of the death benefits for this insurance.

A) 487 B) 497 C) 507 D) 517 E) 527

15. You are given:

(i) $\mathring{e}_{30:\overline{40}|} = 27.692$ (ii) $s(x) = 1 - \frac{x}{\omega}$, $0 \leq x \leq \omega$

(iii) $T(x)$ is the future lifetime random variable for (x).

Calculate $Var(T(30))$.

A) 332 B) 352 C) 372 D) 392 E) 412

16. For a fully discrete 5-payment 10-year decreasing term insurance on (60), you are given:

(i) $b_{k+1} = 1000(10 - k)$, $k = 0, 1, 2, ..., 9$

(ii) Level benefit premiums are payable for five years and equal 218.15 each.

(iii) $q_{60+k} = 0.02 + 0.001k$, $k = 0, 1, 2, ..., 9$

(iv) $i = 0.06$

Calculate ${}_2V$, the benefit reserve at the end of year 2.

A) 70 B) 72 C) 74 D) 76 E) 78

17. You are given:
(i) $T(x)$ and $T(y)$ are not independent.
(ii) $q_{x+k} = q_{y+k} = 0.05$, $k = 0, 1, 2, ...$
(iii) ${}_kp_{xy} = 1.02\, {}_kp_x\, {}_kp_y$, $k = 1, 2, 3, ...$
Into which of the following ranges does $e_{\overline{x:y}}$, the curtate expectation of life of the last survivor status, fall?
A) $e_{\overline{x:y}} \leq 25.7$ B) $25.7 < e_{\overline{x:y}} \leq 26.7$ C) $26.7 < e_{\overline{x:y}} \leq 27.7$
D) $27.7 < e_{\overline{x:y}} \leq 28.7$ E) B) $28.7 < e_{\overline{x:y}}$

18. Subway trains arrive at your station at a Poisson rate of 20 per hour. 25% of the trains are express and 75% are local. The types and number of trains arriving are independent. An express gets you to work in 16 minutes and a local gets you there in 28 minutes. You always take the first train to arrive. Your co-worker always takes the first express. You are both waiting at the same station. Calculate the conditional probability that you arrive at work before your co-worker, given that a local arrives first.
A) 37% B) 40% C) 43% D) 46% E) 49%

19. Beginning with the first full moon in October deer are hit by cars at a Poisson rate of 20 per day. The time between when a deer is hit and when it is discovered by highway maintenance has an exponential distribution with a mean of 7 days. The number hit and the times until they are discovered are independent. Calculate the expected number of deer that will be discovered in the first 10 days following the first full moon in October.
A) 78 B) 82 C) 86 D) 90 E) 94

20. You are given:
(i) $\mu_x(t) = 0.03$, $t \geq 0$ (ii) $\delta = 0.05$
(iii) $T(x)$ is the future lifetime random variable.
(iv) g is the standard deviation of $\overline{a}_{\overline{T(x)|}}$.
Calculate $Pr\left(\overline{a}_{\overline{T(x)|}} > \overline{a}_x - g\right)$.
A) 0.53 B) 0.56 C) 0.63 D) 0.68 E) 0.79

21. (50) is an employee of XYZ Corporation. Future employment with XYZ follows a double decrement model:

(i) Decrement 1 is retirement

(ii) $\mu_{50}^{(1)}(t) = \begin{cases} 0.00 & 0 \le t < 5 \\ 0.02 & 5 \le t \end{cases}$

(iii) Decrement 2 is leaving employment with XYZ for all other causes

(iv) $\mu_{50}^{(2)}(t) = \begin{cases} 0.05 & 0 \le t < 5 \\ 0.03 & 5 \le t \end{cases}$

(v) If (50) leaves employment with XYZ, he will never rejoin XYZ.

Calculate the probability that (50) will retire from XYZ before age 60.

A) 0.069 B) 0.074 C) 0.079 D) 0.084 E) 0.089

22. For a life table with a one-year select period, you are given:

(i)

x	$\ell_{[x]}$	$d_{[x]}$	ℓ_{x+1}	$\mathring{e}_{[x]}$
80	1000	90	–	8.5
81	920	90	–	–

(ii) Deaths are uniformly distributed over each year of age.

Calculate $\mathring{e}_{[81]}$.

A) 8.0 B) 8.1 C) 8.2 D) 8.3 E) 8.4

23. For a fully discrete 3-year endowment insurance of 1000 on (x):

(i) $i = 0.05$ (ii) $p_x = p_{x+1} = 0.7$

Calculate the second year terminal benefit reserve.

A) 526 B) 632 C) 739 D) 845 E) 952

24. You are given: $\mu(x) = \begin{cases} 0.05 & 50 \le x < 60 \\ 0.04 & 60 \le x < 70 \end{cases}$

Calculate ${}_{4|14}q_{50}$.

A) 0.38 B) 0.39 C) 0.41 D) 0.43 E) 0.44

25. For a fully discrete whole life insurance of 1000 on (50), you are given:

(i) The annual per policy expense is 1.

(ii) There is an additional first year expense of 15.

(iii) The claim settlement expense of 50 is payable when the claim is paid.

(iv) All expenses, except the claim settlement expense, are paid at the beginning of the year.

(v) Mortality follows DeMoivre's law with $\omega = 100$.

(vi) $i = 0.05$

Calculate the level expense-loaded premium using the equivalence principle.

A) 27 B) 28 C) 29 D) 30 E) 31

26. For a special fully discrete 5-year deferred whole life insurance of 100,000 on (40), you are given:

(i) The death benefit during the 5-year deferral period is return of benefit premiums paid without interest.

(ii) Annual benefit premiums are payable only during the deferral period.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$ (v) $(IA)^{\,1}_{40:\overline{5}|} = 0.04042$

Calculate the annual benefit premium.

A) 3300 B) 3320 C) 3340 D) 3360 E) 3380

27. You are pricing a special 3-year annuity-due on two independent lives, both age 80. The annuity pays 30,000 if both persons are alive and 20,000 if only one person is alive.

You are given:

(i)

k	${}_kp_{80}$
1	0.91
2	0.82
3	0.72

(ii) $i = 0.05$

Calculate the actuarial present value of this annuity.

A) 78,300 B) 80,400 C) 82,500 D) 84,700 E) 86,800

28. Company ABC sets the contract premium for a continuous life annuity of 1 per year on (x) equal to the single benefit premium calculated using:

(i) $\delta = 0.03$ (ii) $\mu_x(t) = 0.02\,,\ t \geq 0$

However, a revised mortality assumption reflects future mortality improvement and is given by

$$\mu_x(t) = \begin{cases} 0.02 & \text{for } t \leq 10 \\ 0.01 & \text{for } t > 10 \end{cases}$$

Calculate the expected loss at issue for ABC (using the revised mortality assumption) as a percentage of the contract premium.

A) 2% B) 8% C) 15% D) 20% E) 23%

29. A group of 1000 lives each age 30 sets up a fund to pay 1000 at the end of the first year for each member who dies in the first year, and 500 at the end of the second year for each member who dies in the second year. Each member pays into the fund an amount equal to the single benefit premium for a special 2-year term insurance, with:

(i) Benefits:

k	b_{k+1}
0	1000
1	500

(ii) Mortality follows the Illustrative Life Table.

(iii) $i = 0.06$

The actual experience of the fund is as follows:

k	Interest Rate Earned	Number of Deaths
0	0.070	1
1	0.069	1

Calculate the difference, at the end of the second year, between the expected size of the fund as projected at time 0 and the actual fund.

A) 840 B) 870 C) 900 D) 930 E) 960

30. For independent lives (x) and (y), State 1 is that (x) and (y) are alive, and State 2 that (x) is alive but (y) has died, State 3 is the (y) is alive but (x) has died, and State 4 that both (x) and (y) have died.

You are given:

- $s_x(x) = (1 - \frac{x}{100})^{1/2}\,,\ 0 \leq x \leq 100$
- $s_y(y) = 1 - \frac{y}{100}\,,\ 0 \leq y \leq 100$
- $Q_n^{(i,j)}$ is the probability that (x) and (y) are in State j at time $n+1$ given that they are in State i at time n.
- At time 0, (x) is age 54 and (y) is age 75.

Calculate $Q_{10}^{(1,2)}$.

A) Less than 0.026 B) At least 0.026, but less than 0.039

C) At least 0.039, but less than 0.052 D) At least 0.052, but less than 0.065

E) At least 0.065

S. BROVERMAN MLC STUDY GUIDE
PRACTICE EXAM 1 SOLUTIONS

1. The equivalence principle premium is

$1000P_{x:\overline{3}|} = 1000(\frac{1}{\ddot{a}_{x:\overline{3}|}} - d) = 1000(\frac{1}{2.70182} - \frac{.1}{1.1}) = 279.21$.

We are given that (x) dies in the second year. Using the end of the first year as a reference point, there will be the death benefit of 1000 paid one year later (end of the second year) and there will be one premium received just at the start of the second year. ${}_1L$ is the present value, value at the end of the first year, of the insurance payment minus the present value of the future premiums.

This will be ${}_1L = 1000v - 279.21 = 629.88$. Answer: B

2. $\mu_{40}^{(\tau)}(t) = \frac{-\frac{d}{dt}\,{}_tp_{40}^{(\tau)}}{{}_tp_{40}^{(\tau)}}$. ${}_tp_{40}^{(\tau)} = {}_tp_{40}'^{(1)} \cdot {}_tp_{40}'^{(2)} = (1 - \frac{t}{40})(1 - \frac{t}{60}) = 1 - \frac{t}{24} + \frac{t^2}{2400}$

$\frac{-\frac{d}{dt}\,{}_tp_{40}^{(\tau)}}{{}_tp_{40}^{(\tau)}} = \frac{\frac{1}{24} - \frac{2t}{2400}}{1 - \frac{t}{24} + \frac{t^2}{2400}} \rightarrow \mu_{40}^{(\tau)}(20) = \frac{\frac{1}{24} - \frac{400}{2400}}{1 - \frac{20}{24} + \frac{400}{2400}} = .075$. Answer: E

3. The probability that the last death occurs in the 6-th year is ${}_{5|}q_{\overline{35:45}}$.

This can be formulated as ${}_{5|}q_{\overline{35:45}} = {}_6q_{\overline{35:45}} - {}_5q_{\overline{35:45}} = {}_6q_{35} \cdot {}_6q_{45} - {}_5q_{35} \cdot {}_5q_{45}$

(for independent lives, ${}_tq_{\overline{xy}} = {}_tq_x \cdot {}_tq_y$).

From the given information ${}_5q_{35} = .1$, ${}_5q_{45} = .2$.

Also, ${}_6p_{35} = {}_5p_{35} \cdot p_{40} = (.9)(1 - .03) = .873$ and ${}_6p_{45} = {}_5p_{45} \cdot p_{50} = (.8)(1 - .05) = .760.$

Then ${}_{5|}q_{\overline{35:45}} = (1 - .873)(1 - .76) - (.1)(.2) = .01048$. Answer: B

4. We use the equivalence principle relationship

APV expense-loaded premium $=$ APV benefit plus expenses.

$G\ddot{a}_{35} = 100,000A_{35} + .1G\ddot{a}_{35} + 25\ddot{a}_{35} + 250\ddot{a}_{35}$.

Solving for G results in $G = \frac{100{,}000A_{35} + 25\ddot{a}_{35} + 2500\ddot{a}_{35}}{.9G\ddot{a}_{35}} = \frac{100{,}000}{.9}P_{35} + \frac{275}{.9} = 1234.44$.

Answer: D

5. $N(t)$ denotes the Poisson process of the number of glasses of wine drunk in t days; $N(t)$ has parameter $\lambda = 2$ per day. Each glass of wine drunk has a .01 chance of being poisoned. $N_1(t)$ denotes the number of glasses of wine that are poisoned in t days; $N_1(t)$ is a Poisson process with parameter $\lambda_1 = 2(.01) = .02$ per day. The number of glasses of wine that are poisoned in 30 days has a Poisson distribution with a mean of $30(.02) = .6$. The current king will survive at least 30 days if no glasses of wine are poisoned in the next 30 days. The probability of this is $P[N_1(30) = 0] = e^{-.6} = .5488$. Answer: D

6. The variance of the continuous whole life insurance with face amount b is $Var[Z] = b^2[{}^2\overline{A}_x - (\overline{A}_x)^2]$. Since the force of mortality is constant at .02 and $\delta = .04$, $\overline{A}_x = \frac{\mu}{\delta+\mu} = \frac{.02}{.04+02} = \frac{1}{3}$ and ${}^2\overline{A}_x = \frac{\mu}{2\delta+\mu} = \frac{.02}{..08+02} = \frac{1}{5}$, so that $Var[Z] = b^2[\frac{1}{5} - (\frac{1}{3})^2] = \frac{4b^2}{45}$. The single benefit premium is $b\overline{A}_x = b \cdot \frac{1}{3}$. We are told that $b \cdot \frac{1}{3} = \frac{4b^2}{45}$, from which we get $b = 3.75$. Answer: E

7. We assume that we are to find premiums based on the equivalence principle.
We will denote each of the two premiums as Q (assume to be paid the start of each half-year during the first year). The APV of the premiums is $Q[1 + v^{.5} \cdot {}_{.5}p_{30}]$.
The APV of the benefit is $1000vq_{30} + 500v^2\ {}_{1|}q_{30} + 250v^3\ {}_{2|}q_{30}$.
From the Illustrative Table, we have $q_{30} = .00153$, $q_{31} = .00161$ and $q_{32} = .00170$.
Using UDD, the APV of premiums is $Q[1 + v^{.5}(1 - .5(.00153))] = 1.970543Q$.
The APV of the benefit is
$1000v(.00153) + 500v^2(.99847)(.00161) + 250v^3(.99847)(.99839)(.00170) = 2.514466$.
Then $Q = \frac{2.514466}{1.970543} = 1.276$. Answer: A

8. Let q denote the probability of dying within the first two years. Then the number of deaths N in the first two years has a binomial distribution based on $m = 100$ trials and success (dying) probability q. The variance of the binomial is $Var[N] = mq(1-q) = 100q(1-q)$.
q can be formulated as $q = P[\text{die in the 1st year}] + P[\text{survive 1st year and die in 2nd year}]$.
$P[\text{die in the 1st year}] = Q^{(0,2)} = .1$,
$P[\text{survive 1st year and die in 2nd year}]$
$= Q^{(0,0)} \cdot Q^{(0,2)} + Q^{(0,1)} \cdot Q^{(1,2)} = (.7)(.1) + (.2)(.25) = .12$
(this is the combination of staying healthy for the 1st year and dying in the 2nd year, or becoming disabled in the 1st year and dying in the 2nd year).
Therefore, $q = .1 + .12 = .22$ and the $Var[N] = 100(.22)(.78) = 17.16$. Answer: C

9. At the beginning of year 2 the individual is disabled, there are still 2 years left on the 3-year insurance policy. If the individual dies in the 2nd year, there will be a benefit of 1000 paid at that time. The probability of this is $Q^{(2,4)} = .3$ (that is the probability of a disabled individual dying during the year). The APV at time 2 of the death benefit for death in the 2nd year is $1000v(.3) = 285.71$.
The individual can survive the 2nd year and die in the 3rd year, but the benefit will only be payable if the individual is active or disabled at the start of the 3rd year. The probability of remaining disabled to the start of the 3rd year and then dying in the 3rd year is $Q^{(2,2)} \cdot Q^{(2,4)} = (.5)(.3) = .15$. The probability of returning to active as of the start of the 3rd year and then dying in the 3rd year is $Q^{(2,1)} \cdot Q^{(1,4)} = (.2)(.1) = .02$. The combined probability of surviving to the start of the 3rd year and not withdrawing, and then dying in the 3rd year, is $.15 + .02 = .17$. The APV at the beginning of the 2nd year of the death benefit for death in the 3rd year is $1000v^2(.17) = 154.20$. The total APV of the death benefit is $285.71 + 154.20 = 439.91$. Answer: A

10. The initial benefit reserve for policy year 10 is ${}_9V + P = 343$ (where P is the benefit premium). The net amount at risk for policy year 10 is $b - {}_{10}V = 872$.
Using the net amount at risk form of the recursive relationship for benefit reserve, for year 10, we have $({}_9V + P)(1+i) - (b - {}_{10}V)q_{x+9} = {}_{10}V$, which becomes $(343)(1.03) - (872)(.02904) = {}_{10}V \rightarrow {}_{10}V = 328$.
Then, $b = {}_{10}V + 872 = 1200$. Then $P = 1200P_x = 1200(\frac{1}{\ddot{a}_x} - d) = 46.91$, so that ${}_9V = 343 - P = 296.1$. Answer: C

11. The level annual premium based on the equivalence principle is
$1000P_{x:\overline{2}|} = 1000(\frac{1}{\ddot{a}_{x:\overline{2}|}} - d)$, where $\ddot{a}_{x:\overline{2}|} = 1 + vp_x$.
If we find p_x we can find the premium.
A key point in solving the problem is $A_{x:\overline{2}|} = vq_x + v^2p_x = v - (v - v^2)p_x$.
This follows from $A_{x:\overline{2}|} = A^{1}_{x:\overline{2}|} + v^2\,{}_2p_x = vq_x + v^2\,{}_{1|}q_x + v^2\,{}_2p_x$
$= vq_x + v^2p_xq_{x+1} + v^2p_xp_{x+1} = vq_x + v^2p_x(q_{x+1} + p_{x+1}) = vq_x + v^2p_x$.
We are given $668 + 258vp_x = 1000A_{x:\overline{2}|} = 1000(v - (v - v^2)p_x)$.
Using $v = 1 - d = .94$, we solve for $p_x = .9099$.
Then $\ddot{a}_{x:\overline{2}|} = 1 + vp_x = 1.8553$, and $1000P_{x:\overline{2}|} = 1000(\frac{1}{1.8553} - .06) = 479$. Answer: B

12. We are given $100,000(IA)^{\,1}_{41:\overline{10}|} = 16,736$. We wish to find $100,000(IA)^{\,1}_{40:\overline{10}|}$.

We use the relationship $(IA)^{1}_{x:\overline{n}|} = A^{1}_{x:\overline{n}|} + vp_x\,(IA)^{\,1}_{x+1:\overline{n}|} - nv^{n+1}\,{}_{n|}q_x$. This can be seen by looking at the time line of possible death benefit payments; the first row is the sum of the second and third rows.

	x	$x+1$	$x+2$	$x+3$	$\ldots$	$x+n-1$	$x+n$	$x+n-1$
$(IA)^{1}_{x:\overline{n}|}$	←	1	2	3		$n-1$	n	
$A^{1}_{x:\overline{n}|}$	←	1	1	1		1	1	
$vp_x\,(IA)^{\,1}_{x+1:\overline{n-1}|}$		←	1	2		$n-2$	$n-1$	n
$-\,nv^{n+1}\,{}_{n|}q_x$								$-\,n$

Then, $(IA)^{\,1}_{40:\overline{10}|} = A^{\,1}_{40:\overline{10}|} + vp_{40}\,(IA)^{\,1}_{41:\overline{10}|} - 10v^{11}\,{}_{10|}q_{40}$

$A^{\,1}_{40:\overline{10}|} = A_{40} - v^{10}\,{}_{10}p_{40}A_{50} = (.16132) - (.53667)(.24905) = .027662$

(we use values from the Illustrative Table, and notice that $v^{10}\,{}_{10}p_{40} = {}_{10}E_{40}$).

Also $v^{11}\,{}_{10|}q_{40} = v \cdot v^{10}\,{}_{10}p_{40} \cdot q_{50} = \frac{1}{1.06} \cdot (.53667)(.00592) = .002997$.

Then $(IA)^{\,1}_{40:\overline{10}|} = .027662 + \frac{1}{1.06} \cdot (1 - .00278)(.16736) - 10(.002997) = .1551$.

We then multiply by 100,000 to get $100,000(IA)^{\,1}_{40:\overline{10}|} = 155,510$. Answer: D

13. Assuming a starting asset share of 0, the accumulation of asset share in the 1st year is
$[0 + 100(.6)](1.1) - 1000q_x = p_x \cdot {}_1AS$.

If we knew the value of q_x, we could find ${}_1AS$.

Using the recursive relationship for benefit reserve for the 1st year, we have
$[0 + 80](1.1) - 1000q_x = p_x(40)$, and solving for q_x results in $q_x = .05$.

Then $60(1.1) - 1000(.05) = (.95) \cdot {}_1AS \rightarrow {}_1AS = 16.84$. Answer: A

14. The APV of the death benefit is $10,000[vq_{40}^{(1)} + v^2\,{}_{1|}q_{40}^{(1)} + v^3\,{}_{2|}q_{40}^{(1)}]$.

Since decrement 2 occurs at the end of the year, for each year, $q_x^{(1)} = q_x^{\prime(1)}$, and
$q_x^{(2)} = (1 - q_x^{\prime(1)}) \cdot q_x^{\prime(2)}$.

For decrement 1, we have $q_x^{(1)} = q_x^{\prime(1)} = 1 - e^{-.02} = .01980$ for $x = 40,\ 41,\ 42$, since the force of decrement is constant. Also $p_x^{(\tau)} = p_x^{\prime(1)} \cdot p_x^{\prime(2)} = e^{-.02} \cdot (.96) = .94099$ for $x = 40,\ 41.$

Then, ${}_{1|}q_{40}^{(1)} = p_{40}^{(\tau)} \cdot q_{41}^{(1)} = (.94099)(.01980) = .01863$ and
${}_{2|}q_{40}^{(1)} = {}_2p_{40}^{(\tau)} \cdot q_{42}^{(1)} = p_{40}^{(\tau)} \cdot p_{41}^{(\tau)} \cdot q_{42}^{(1)} = (.94099)^2(.01980) = .01753$.

The APV of the death benefit is
$10,000[v(.01980) + v^2(.01863) + v^3(.01753)] = 506.71$. Answer: C

15. From the form of $s(x)$ we see that survival from birth follows DeMoivre's Law with upper age ω. Once ω is found, $Var[T(30)] = \frac{(\omega-30)^2}{12}$. Under DeMoivre's Law, ${}_tp_x = 1 - \frac{t}{\omega-x}$.
$\mathring{e}_{30:\overline{40|}} = \int_0^{40} {}_tp_{30}\,dt = \int_0^{40} [1 - \frac{t}{\omega-30}]\,dt = 40 - \frac{40^2}{2(\omega-30)} = 27.692$.
Solving for ω results in $\omega = 95$. Then $Var[T(30)] = \frac{(95-30)^2}{12} = 352.1$. Answer: B

16. We use the recursive reserve formula, $({}_kV + P)(1+i) - b_{k+1}\cdot q_{x+k} = p_{x+k}\cdot {}_{k+1}V$.
Also, ${}_0V = 0$ for benefit reserves. For the first year, we have
$(0 + 218.15)(1.06) - 10,000(.02) = .98\cdot {}_1V \rightarrow {}_1V = 31.88$.
For the second year, we have
$(31.88 + 218.15)(1.06) - 9,000(.021) = .979\cdot {}_2V \rightarrow {}_2V = 77.66$. Answer: E

17. $e_{\overline{xy}} = e_x + e_y - e_{xy}$ is a valid relationship for all survival distributions of $T(x)$ and $T(y)$, whether or not they are independent. Since $q_{x+k} = .05$ for al k, it follows that $p_{x+k} = .95$ for all k, and then $e_x = \sum_{t=1}^{\infty} {}_tp_x = \sum_{t=1}^{\infty}(.95)^t = \frac{.95}{1-.95} = 19$. This follows from the fact that
${}_tp_x = p_x\cdot p_{x+1}\cdots p_{x+t-1} = (.95)^t$ and $1 + r + r^2 + r^3 + \cdots = \frac{1}{1-r}$.
Also, $e_y = 19$, since $q_{y+k} = .05$ for all k.
$e_{xy} = \sum_{t=1}^{\infty} {}_tp_{xy} = \sum_{t=1}^{\infty} 1.02(.95)^{2t} = 1.02\cdot\frac{(.95)^2}{1-(.95)^2} = 9.44$.
Then $e_{\overline{xy}} = 19 + 19 - 9.44 = 28.56$. Answer: D

18. Given that a local train arrives first, you will get to work 28 minutes after that local train arrives, since you will take it. Your co-worker will wait for first express train. You will get to work before your co-worker if the next express train (after the local) arrives more than 12 minutes after the local. We expect 5 express trains per hour, so the time between express trains is exponentially distributed with a mean of $\frac{1}{5}$ of an hour, or 12 minutes. Because of the lack of memory property of the exponential distribution, since we are given that the next train is local, the time until the next express train after that is exponential with a mean of 12 minutes. Therefore, the probability that after the local, the next express arrives in more than 12 minutes is $P[T > 12]$, where T has an exponential distribution with a mean of 12. This probability is $e^{-12/12} = e^{-1} = .368$ (37%). Answer: A

19. We expect a deer to be hit by a car every $\frac{1}{20} = .05$ days. For the deer that is expected to be hit at $.05k$ days, the chance of being discovered within the first 10 days is the probability of being discovered within $10 - .05k$ days after being hit. Since time of discovery after being hit has an exponential distribution with mean 7 days, this probability is $1 - e^{-(10-.05k)/7}$ (the prob. $P[T < 10 - .05k]$, where T is exponential with mean 7). The expected number of deer discovered within 10 days following the first full moon in October is $\sum_{k=1}^{199} [1 - e^{-(10-.05k)/7}]$, since each term in the sum is the expected number of deer discovered for the one deer hit at time k. The sum goes to 199 since the 200-th deer is expected to be hit just at time 10, and cannot be discovered before time 10.

$\sum_{k=1}^{199} [1 - e^{-(10-.05k)/7}] = 199 - e^{-10/7} \cdot e^{.05/7}[1 + e^{.05/7} + (e^{.05/7})^2 + \cdots + (e^{.05/7})^{198}]$

$= 199 - e^{-9.95/7} \cdot \frac{(e^{.05/7})^{199} - 1}{e^{.05/7} - 1} = 93.2$ (round up to the next integer value 94) . Answer: E

20. With constant force of mortality $\mu = .03$ and force of interest $\delta = .05$, $\overline{a}_x = \frac{1}{\delta + \mu} = 12.5$.

The variance of $\overline{a}_{\overline{T(x)}|}$ is

$\frac{1}{\delta^2}[{}^2\overline{A}_x - (\overline{A}_x)^2] = \frac{1}{\delta^2}[\frac{\mu}{2\delta + \mu} - (\frac{\mu}{\delta + \mu})^2] = \frac{1}{(.05)^2}[\frac{.03}{.1 + .03} - (\frac{.03}{.05 + .03})^2] = 36.06$.

The standard deviation is $\sqrt{36.06} = 6.00$.

We wish to find $P[\overline{a}_{\overline{T(x)}|} > 12.5 - 6.00] = P[\overline{a}_{\overline{T(x)}|} > 6.5]$.

We solve for n, from the equation $\overline{a}_{\overline{n}|} = 6.5$, so that $\frac{1 - e^{-.05n}}{.05} = 6.5$, so that $e^{-.05n} = .675$ (which is equivalent to $n = -\frac{ln(.675)}{.05} = 7.86$ years).

If (x) dies exactly at that time n, then $\overline{a}_{\overline{n}|} = 6.5$, so it follows that $P[\overline{a}_{\overline{T(x)}|} > 6.5]$ is equal to $P[T(x) > n] = {}_np_x = e^{-\mu n} = e^{-.03n}$ (the present value of the annuity is > 6.5 if (x) lives at least n years). Since $e^{-.05n} = .675$, it follows that $e^{-.03n} = (e^{-.05n})^{3/5} = (.675)^{.6} = .790$.

Therefore, $P[\overline{a}_{\overline{T(x)}|} > 6.5] = .790$. Answer: E

21. The probability is ${}_{10}q_{50}^{(1)}$, which can be written as ${}_5q_{50}^{(1)} + {}_5p_{50}^{(\tau)} \cdot {}_5q_{55}^{(1)}$.

Since the force of decrement for retirement is 0 to age 55, ${}_5q_{50}^{(1)} = 0$.

Also, for $0 \leq t < 5$, $\mu_{50}^{(\tau)}(t) = \mu_{50}^{(1)}(t) + \mu_{50}^{(2)}(t) = .05$, so that ${}_5p_{50}^{(\tau)} = e^{-.05(5)} = .77880$.

For $0 \leq t < 5$, $\mu_{55}^{(\tau)}(t) = \mu_{55}^{(1)}(t) + \mu_{55}^{(2)}(t) = .05$, so that ${}_tp_{55}^{(\tau)} = e^{-.05t}$ and

${}_5q_{55}^{(1)} = \int_0^5 {}_tp_{55}^{(\tau)} \cdot \mu_{55}^{(1)}(t)\, dt = \int_0^5 e^{-.05t}(.02)\, dt = (.02) \cdot \frac{1 - e^{-.25}}{.05} = .08848$.

Then, ${}_{10}q_{50}^{(1)} = {}_5q_{50}^{(1)} + {}_5p_{50}^{(\tau)} \cdot {}_5q_{55}^{(1)} = 0 + (.77880)(.08848) = .0689$. Answer: A

22. With a one-year select period, $\overset{\circ}{e}_{[81]+1} = \overset{\circ}{e}_{82}$, so that
$\overset{\circ}{e}_{[81]} = \overset{\circ}{e}_{[81]:\overline{1}|} + p_{[81]} \cdot \overset{\circ}{e}_{82} = \int_0^1 {}_tp_{[81]}\,dt + p_{[81]} \cdot \overset{\circ}{e}_{82} = \int_0^1 (1 - {}_tq_{[81]})\,dt + p_{[81]} \cdot \overset{\circ}{e}_{82}$
$= \int_0^1 (1 - t \cdot q_{[81]})\,dt + p_{[81]} \cdot \overset{\circ}{e}_{82} = \int_0^1 (1 - \frac{90}{920}t)\,dt + (\frac{90}{920}) \cdot \overset{\circ}{e}_{82} =$
(using UDD and $q_{[81]} = \frac{90}{1000} = .09$).
From the table we have $\ell_{[80]+1} = \ell_{81} = 910$ and $\ell_{[81]+1} = \ell_{82} = 830$, so that $p_{81} = \frac{830}{910}$
and $q_{81} = \frac{8}{91}$.
Then, we use the relationship $\overset{\circ}{e}_{[80]} = \overset{\circ}{e}_{[80]:\overline{1}|} + p_{[80]} \cdot \overset{\circ}{e}_{81:\overline{1}|} + {}_2p_{[80]} \cdot \overset{\circ}{e}_{82}$ to solve for $\overset{\circ}{e}_{82}$.
From UDD we have $\overset{\circ}{e}_{[80]:\overline{1}|} = \int_0^1 (1 - .09t)\,dt = .955$ and $\overset{\circ}{e}_{81:\overline{1}|} = \int_0^1 (1 - \frac{8}{91}t)\,dt = .956$,
$p_{[80]} = .91$, ${}_2p_{[80} = p_{[80]} \cdot p_{81} = (.91)(\frac{83}{91}) = .83$. Then
$8.5 = .955 + (.91)(.956) + (.83)\overset{\circ}{e}_{82}$, so that $\overset{\circ}{e}_{82} = 8.04$.
Finally $\overset{\circ}{e}_{[81]} = \overset{\circ}{e}_{[81]:\overline{1}|} + p_{[81]} \cdot \overset{\circ}{e}_{82} = .951 + (\frac{830}{920}) \cdot (8.04) = 8.20$. Answer: C

23. The 2nd year terminal reserve for a 3-year endowment insurance can be formulated as

$${}_2V_{x:\overline{3}|} = 1 - \frac{\ddot{a}_{x+2:\overline{1}|}}{\ddot{a}_{x:\overline{3}|}} ,$$

where $\ddot{a}_{x+2:\overline{1}|} = 1$ and $\ddot{a}_{x:\overline{3}|} = 1 + vp_x + v^2\,{}_2p_x = 1 + \frac{.7}{1.05} + \frac{(.7)^2}{(1.05)^2} = 2.11$.
Then ${}_2V_{x:\overline{3}|} = 1 - \frac{1}{2.11} = .526$. For face amount 1000, the reserve is 526. Answer: A

24. ${}_{4|14}q_{50} = {}_4p_{50} - {}_{18}p_{50}$.
${}_4p_{50} = e^{-\int_0^4 \mu(50+t)\,dt} = e^{-(.05)(4)} = .81873$.
${}_{18}p_{50} = {}_{10}p_{50} \cdot {}_8p_{60} = e^{-\int_0^{10} \mu(50+t)\,dt} \cdot e^{-\int_0^8 \mu(60+t)\,dt} = e^{-.05(10)} \cdot e^{-.04(8)} = .44043$.
We must split the probability because of the change in force of mortality at age 60.
${}_{4|14}q_{50} = .81873 - .44043 = .3783$. Answer: A

25. APV expense-loaded premium = APV benefit plus expense
$G\ddot{a}_{50} = 1000A_{50} + 15 + \ddot{a}_{50} + 50A_{50}$.
From DeMoivre's law, $A_x = \frac{1}{\omega - x} \cdot a_{\overline{\omega - x}|}$, so that
$A_{50} = \frac{1}{100-50} \cdot a_{\overline{100-50}|} = \frac{1}{50} \cdot \frac{1-v^{50}}{i} = .36512$.
Then $\ddot{a}_{50} = \frac{1-A_{50}}{d} = \frac{1-.36512}{.05/1.05} = 13.332$.
The premium equation becomes $13.332G = 1000(.36512) + 15 + 13.332 + 50(.36512)$,
so that $G = 30.88$. Answer: E

26. If the annual benefit premium is Q, then $Q \cdot \ddot{a}_{40:\overline{5}|} = Q(IA)^{\ \ 1}_{40:\overline{5}|} + 100,000 \cdot {}_{5|}A_{40}$.

To find $\ddot{a}_{40:\overline{5}|}$, we use the relationship $\ddot{a}_{40} = \ddot{a}_{40:\overline{5}|} + {}_5E_{40} \cdot \ddot{a}_{45}$. Using values from the Illustrative Table, we have $14.8166 = \ddot{a}_{40:\overline{5}|} + (.73529)(14.1121)$, so that $\ddot{a}_{40:\overline{5}|} = 4.4401$.
Also, ${}_{5|}A_{40} = {}_5E_{40} \cdot A_{45} = (.73529)(.20120) = .14794$.
Then $4.4401Q = .04042Q + 100,000(.14794)$, so that $Q = 3363$. Answer: D

27. The APV of the annuity can be formulated as an annuity of 20,000 per year while at least one is alive, combined with an additional 10,000 per year while both are alive (this makes for a total payment of 30,000 per year while both are alive). The APV is

$$20,000\ddot{a}_{\overline{xy}:\overline{3}|} + 10,000\ddot{a}_{xy:\overline{3}|} = 20,000(\ddot{a}_{x:\overline{3}|} + \ddot{a}_{y:\overline{3}|} - \ddot{a}_{xy:\overline{3}|}) + 10,000\ddot{a}_{xy:\overline{3}|}$$
$$= 20,000(\ddot{a}_{x:\overline{3}|} + \ddot{a}_{y:\overline{3}|}) - 10,000\ddot{a}_{xy:\overline{3}|} .$$

Both x and y are 80. $\ddot{a}_{80:\overline{3}|} = 1 + vp_{80} + v^2\,{}_2p_{80} = 2.61043$ and
$\ddot{a}_{80:80:\overline{3}|} = 1 + vp_{80:80} + v^2\,{}_2p_{80:80}$. Since the two lives are independent, we have
$p_{80:80} = p_{80} \cdot p_{80} = (.91)^2$, and ${}_2p_{80:80} = {}_2p_{80} \cdot {}_2p_{80} = (.82)^2$, so that
$\ddot{a}_{80:80:\overline{3}|} = 2.39855$.
The APV of the annuity is $20,000(2.61042 \times 2) - 10,000(2.39855) = 80,431$. Answer: B

28. The loss at issue is the PVRV (present value random variable) of benefit to be paid minus the PVRV of premium to be received. Since this is a single premium policy, the premium received is a single payment equal to the contract premium. Based on the original mortality assumption, the contract premium is $\overline{a}_x = \frac{1}{\delta+\mu} = \frac{1}{.03+.02} = 20$ (the is the continuous annuity value for a constant force of mortality). The loss at issue is $L = Y - 20$, where Y is the PVRV of the continuous life annuity. The expected loss at issue will be $E[L] = E[Y] - 20$, where $E[Y]$ is calculated based on the revised mortality assumption.

$$E[Y] = \overline{a}_x = \overline{a}_{x:\overline{10}|} + v^{10}\,{}_{10}p_x \cdot \overline{a}_{x+10}$$
$$= \int_0^{10} e^{-.03t}e^{-.02t}\,dt + e^{-.03(10)}e^{-.02(10)} \cdot \frac{1}{.03+.01}$$
$$= \frac{1-e^{-.05(10)}}{.05} + e^{-.05(10)} \cdot \frac{1}{.03+.01} = 23.03 .$$

We have split the whole life annuity into age intervals over which the force of mortality is constant. Then $E[L] = 23.03 - 20 = 3.03$, which is 15% of the contract premium.
Answer: C

29. The single benefit premium per person is $1000vq_{30} + 500v^2\ {}_{1|}q_{30}$.
Using the Illustrative Table, we have $q_{30} = .00153$ and ${}_{1|}q_{30} = p_{30} \cdot q_{31} = .0016075$.
The single benefit premium is 2.15873 . Since this is a single benefit premium, the expected size of the fund at the end of the 2-year term is 0. The actual fund progresses in the following way.
$1000(2.15873) = 2158.73$ is collected at the start of the first year.
At the end of the first year, with interest minus death benefit, the fund value is
$2158.73(1.07) - 1000 = 1309.84$.
With interest minus death benefits to the end of the second year, the fund value is
$1309.84(1.069) - 500 = 900.2$. Answer: C

30. $Q_{10}^{(1,2)}$ is the probability that for (x) and (y), alive at age 64 and 85, (x) will survive the year, but (y) will die within the year. This probability that (64) survives the year is p_{64} for (x), which is $\frac{s_x(65)}{s_x(64)} = .986013$. The probability that (85) dies during the year is q_{85} for (y), which is
$1 - \frac{s_y(86)}{s_y(85)} = .06667$ ((y)'s mortality follows DeMoivre's Law with $\omega = 100$).
Then, $Q_{10}^{(1,2)} = p_{64}^{x} \cdot q_{85}^{y} = (.986013)(.06667) = .066$. Answer: E