EXAM C QUESTIONS OF THE WEEK

S. Broverman, 2005

Question 9 - Week of September 19

A portfolio of insurance policies is made up of two classes of equal size.
For a policy in Class A, the size of the annual loss has a continuous uniform distribution on the interval $(0, 100)$. For a policy in Class B, the size of the annual loss has a continuous uniform distribution on the interval $(0, 200)$. A policy is chosen at random from the portfolio.

(a) Suppose the annual loss is 50.
(i) Find the probability that the policy is in Class A.
(ii) Find the expected value of the next annual loss from the same policy.
(iii) Find the probability that the next annual loss from the same policy will be less than 100.

(b) Suppose that the annual loss is greater than 50. Find the pdf for the distribution of the next annual loss from the same policy. Verify that it is a properly defined pdf.

The solution can be found below.

Question 9 Solution

(a)(i) $P[A|X=50] = \frac{P[50|A]\cdot P[A]}{P[50|A]\cdot P[A]+P[50|B]\cdot P[B]} = \frac{(.01)(.5)}{(.01)(.5)+(.005)(.5)} = \frac{2}{3}$.

(ii) $E[X_2|X_1=50] = E[X_2|A]\cdot P[A|X_1=50] + E[X_2|B]\cdot P[B|X_1=50]$
$= (50)(\frac{2}{3}) + (100)(\frac{1}{3}) = \frac{200}{3}$.

(iii) $P[X_2<100|X_1=50] = \frac{P[X_1=50\cap X_2<100]}{P[X_1=50]}$
$P[X_1=50] = P[50|A]\cdot P[A] + P[50|B]\cdot P[B] = .0075$,
$P[X_1=50\cap X_2<100]$
$= P[X_1=50\cap X_2<100|A]\cdot P[A] + P[X_1=50\cap X_2<100|B]\cdot P[B]$
$= (.01)(1)(.5) + (.005)(.5)(.5) = .00625$.
$P[X_2<100|X_1=50] = \frac{P[X_1=50\cap X_2<100]}{P[X_1=50]} = \frac{.00625}{.0075} = \frac{5}{6}$

(b) $f(x_2|x_1>50) = \frac{f(50,x_2)}{1-F(50)} = \frac{P[X_1>50\cap X_2=x_2]}{1-F(50)}$
$P[X_1>50] = P[X_1>50|A]\cdot P[A] + P[X_1>50|B]\cdot P[B] = .625$
$P[X_1>50\cap X_2=x_2]$
$= P[X_1>50\cap X_2=x_2|A]\cdot P[A] + P[X_1>50\cap X_2=x_2|B]\cdot P[B]$
$= [1-F_A(50)]f_A(x_2)(.5) + [1-F_B(50)]f_B(x_2)(.5)$
If $x<100$, then $F_A(x) = \frac{x}{100}$ and if $x\geq 100$ then $F_A(x)=1$.
$F_B(x) = \frac{x}{200}$ for all x in $(0,200)$.
If $x_2<100$, then
$P[X_1>50\cap X_2=x_2] = (.5)(.01)(.5) + (.75)(.005)(.5) = .004375$,and
$f(x_2|x_1>50) = \frac{.004375}{.625} = .007$.
If $x_2\geq 100$, then
$P[X_1>50\cap X_2=x_2] = (0) + (.75)(.005)(.5) = .001875$,and
$f(x_2|x_1>50) = \frac{.001875}{.625} = .003$.

$\int_0^{200} f(x_2|x_1>50)\,dx_2 = \int_0^{100}(.007)\,dx_2 + \int_{100}^{200}(.003)\,dx_2 = 1$.