EXAM M QUESTIONS OF THE WEEK

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Question 7 - Week of September 5

You are given the following table of probabilities in a 3-decrement model.

x	$q_x^{(1)}$	$q_x^{(2)}$	$q_x{}^{(3)}$
30	.010	.015	.020
31	.025	.040	.060
32	.025	.100	.050

(a) Calculate each of the following probabilities

$${}_3q_{30}^{(2)} \qquad {}_{1|}q_{30}^{(\tau)} \qquad {}_3p_{30}^{(\tau)}$$

(b) Using an annual effective interest rate of 6%, find the actuarial present value at age 30 for a 3-year discrete term benefit that pays 1000 at the end of the year of decrement if due to decrement 1.

(c) Show that the data in this table is inconsistent with the independence of random variables $T(30)$ and J.

The solution can be found below.

Question 7 Solution

(a) ${}_3q_{20}^{(2)} = q_{30}^{(2)} + p_{30}^{(\tau)} \cdot q_{31}^{(2)} + {}_2p_{30}^{(\tau)} \cdot q_{32}^{(2)}$
$= (.015) + (.955)(.040) + (.955)(.875)(.100) = .13676$.

${}_{1|}q_{30}^{(\tau)} = p_{30}^{(\tau)} \cdot q_{31}^{(\tau)} = (.955)(.125) = .119375$

${}_3p_{30}^{(\tau)} = p_{30}^{(\tau)} \cdot p_{31}^{(\tau)} \cdot p_{32}^{(\tau)} = (.955)(.875)(.825) = .68939$

(b) APV $= 1000[vq_{30}^{(1)} + v^2 \cdot {}_{1|}q_{30}^{(1)} + v^3 \cdot {}_{2|}q_{30}]$
$= 1000[\frac{.01}{1.06} + \frac{(.955)(.025)}{(1.06)^2} + \frac{(.955)(.875)(.025)}{(1.06)^3}] = 48.22$.

(c) If T and J are independent, then ${}_nq_x^{(j)} = {}_nq_x^{(\tau)} \cdot f_J(j)$ for all n and j.
Thus, if T and J are independent then
$q_{30}^{(1)} = q_{30}^{(\tau)} \cdot f_J(1) \rightarrow .010 = (.045) \cdot f_J(1) \rightarrow f_J(1) = \frac{.010}{.045} = \frac{2}{9}$, and
$q_{31}^{(1)} = q_{31}^{(\tau)} \cdot f_J(1) \rightarrow .025 = (.125) \cdot f_J(1) \rightarrow f_J(1) = \frac{.025}{.125} = \frac{1}{5}$.
The inconsistent values of $f_J(1)$ indicate that T and J cannot be independent.