

EXAM P QUESTIONS OF THE WEEK

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Question 7 - Week of September 5

A production process for electronic components has followup a inspection procedure. Inspectors assign a rating of high, medium or low to each component inspected. Long run inspection data have yielded the following probabilities for component ratings:

$$P(\text{high}) = .5, P(\text{medium}) = .4, P(\text{low}) = .1.$$

Find the probability that in the next batch of 5 components inspected, at least 3 are rated high, and at most 1 is rated low.

- A) Less than .26 B) At least .26 but less than .32 C) At least .32 but less than .38
D) At least .38 but less than .44 E) At least .44

The solution can be found below.

Question 7 Solution

We can use the multinomial distribution. For a batch of n components tested, the probability that a are rated high, b are rated medium and c are rated low is $\frac{n!}{a!b!c!} (.5)^a (.4)^b (.1)^c$.

In a batch of 5, in order to have at least 3 high and at most 1 low, the following combinations are possible 5H, 4H and 1M, 4H and 1L, 3H and 2M, and 3H and 1M and 1L.

The probabilities of these combinations are

$$P(5H) = \frac{5!}{5!0!0!} (.5)^5 (.4)^0 (.1)^0 = .03125 ,$$

$$P(4H \text{ and } 1M) = \frac{5!}{4!1!0!} (.5)^4 (.4)^1 (.1)^0 = .125 ,$$

$$P(4H \text{ and } 1L) = \frac{5!}{4!0!1!} (.5)^4 (.4)^0 (.1)^1 = .03125 ,$$

$$P(3H \text{ and } 2M) = \frac{5!}{3!2!0!} (.5)^3 (.4)^2 (.1)^0 = .20 ,$$

$$P(3H \text{ and } 1M \text{ and } 1L) = \frac{5!}{3!1!1!} (.5)^3 (.4)^1 (.1)^1 = .1 .$$

The total probability is $.03125 + .125 + .03125 + .20 + .1 = .4875$.

Answer: E